# Math 187 Test I

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The test will begin at 1:40 pm and end at 2:35 pm (halfway through the interclass period). You may use a calculator without graphing or symbolic computation capability. Cell phones and PDA's must be turned off and out of sight.

The values of the questions are supposed equal for the moment (though this may be modified based on class performance). You may skip one of problems 5,6,7 (whose values are definitely equal for this reason); if you do all three your best work will count. You must do problems 1,2,3,4, and 8.

There is no particular reason to do the problems in order: if you find yourself staring at one problem for too long, go to a different one!

1. Consider the conditional sentence "If the sun is shining, there are lots of people at the beach".

   State the converse of this statement (as an English sentence).

   State the contrapositive of this statement (as an English sentence).

   One of these sentences (the converse or the contrapositive) is equivalent to the original conditional. Write it down. You do not need to prove the equivalence.

   One of these sentences (the converse or the contrapositive) is not equivalent to the original conditional. Describe a situation in which this sentence is false but the original conditional sentence is true.

2. Use a truth table to demonstrate that $(P \vee Q) \rightarrow R$ is logically equivalent to $(P \rightarrow R) \wedge (Q \rightarrow R)$. Be sure to show enough columns to indicate all of your work, and to highlight significant columns and say something about them in English.

3. Venn diagrams

   (a) Illustrate the identity

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

   using Venn diagrams. Clearly label all sets in your diagrams, using keys to shadings or colors when appropriate. Clearly indicate which set in each diagram is the final result.

(b) The equation

$$A - (B \cup C) = (A - B) \cup (A - C)$$

is not an identity. Find finite sets $A$, $B$, $C$ for which this equation is not true (and show by calculation that it is not true for these sets). You can use a Venn diagram to help find a counterexample, as we discussed in class.

4. A state has license plates consisting of four letters followed by three digits.

   You need to set up calculations for your answers to this problem, but if you don't have a calculator (or if the number is too large for your calculator) the set-up by itself is enough.

   (a) If no additional conditions are imposed, how many license plates are possible?

   (b) How many license plates are possible if no letter can appear more than once and no digit can be followed immediately by the same digit?

   (c) How many license plates are possible if repetitions of letters and digits are allowed and at least one A *or* at least one 8 appears on the plate? Hint: think about plates which contain neither an A nor an 8.

5. Apply de Morgan's laws and the double negation rule to convert

$$\neg((P \wedge \neg Q) \vee R)$$

to a form in which negation ($\neg$) is applied only to single letters. Show the two or three stages of your calculation.

6. Sets and set notation

(a) Write the set $\{x \in \mathcal{Z} \mid 4|x \wedge x|16\}$ in the notation which lists all its elements.

(b) Write the set $\{x \mid x \subseteq \{1, 2, 3\} \wedge |x| \geq 2\}$ in the notation which lists all of its elements. Hint: this set has four elements, and the elements are sets, not numbers.

(c) Consider the two statements "for all sets $A$, $B$, and $C$, if $A \in B$ and $B \in C$, then $A \in C$" and "for all sets $A$, $B$, and $C$, if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$". One of these statements is true: tell me which one (you do not need to prove it). Give examples of sets $A$, $B$, $C$ which are a counterexample to the other statement.

7. Consider the statement "for every integer $x$ there is an integer $y$ such that $y > x$".

   Write this statement using quantifier notation: no English words at all should appear in what you write.

   Write the negation of the statement in a form with the negation as far to the right as possible (in other words, $\neg$ should not appear in front of a quantifier). (You are permitted to use the fact that $\neg y > x$ is equivalent to $y \leq x$, which will give you a form with no appearance of $\neg$ at all.)

   Translate the negation into a natural sentence of mathematical English.

8. Prove the theorem "The sum of two odd integers is even" in the style we have used in class. Begin by rewriting the theorem to be proved as a conditional. Be sure to use the definitions of "odd" and "even".