Math 187 Final Exam

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This exam begins at 1 pm and ends officially at 3 pm. If everyone agrees, the exam will be extended until 3:15 pm. You may use your book and one notebook-sized sheet of notes, as announced on the final review document. You may use a scientific calculator with no graphing or symbolic computation capability.

There are 10 questions on this exam. The question you do worst on (or omit) will be dropped. Your final exam and course grades will be posted on my web site by the ID number found on the first page of your paper. Good luck and have a good holiday season.

1. Show that
$$P \rightarrow (Q \rightarrow R)$$
and
$$(P \wedge Q) \rightarrow R$$
are logically equivalent using the method of truth tables. Be sure to comment on relevant facts about columns in your tables.

Show that $(P \vee Q) \rightarrow R$ and $(P \rightarrow R) \vee (Q \rightarrow R)$ are *not* logically equivalent by giving a counterexample (an assignment of truth values to the letters which makes their values different).

2. Two permutations of $\{1, 2, 3, 4, 5\}$ are given, one in array notation and one in cycle notation. Compute $\sigma \circ \tau$ and τ^{-1}; you may use either cycle or array notation to state your answers.

$$\sigma = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 5 & 4 \end{bmatrix}$$

$$\tau = (1\,3\,5)(2\,4)$$

3. The license plate numbers of a certain state consist of three letters and four digits. State how many plates are possible under each set of additional conditions. You may leave your answer set up in terms of standard arithmetic operations and binomial coefficients (no final calculator answer is required in this problem).

   (a) The letters appear before the digits; there are no other restrictions.

   (b) The letters appear before the digits; no letter or digit can appear more than once.

   (c) The letters appear before the digits; repetitions are allowed; no E or 5 appears on the plate.

   (d) The letters and digits are mixed; no letter or digit appears more than once; the letters appear in alphabetical order and the digits in numerical order. A1B34E5 is an allowed plate, for example. Hint: think binomial coefficients (but carefully).

4. 28 children are unleashed at a Sunday buffet. 14 of them have waffles. 13 of them have pancakes. 20 of them have French toast. 6 of them have waffles and pancakes. 7 of them have waffles and French toast. 10 of them have pancakes and French toast. Determine how many of them had all three. Your work should be set up using the Inclusion/Exclusion Principle (so I can tell that you know it).

5. I am sent by my children to get a dozen bagels (there are 13 bagels in a baker's dozen, of course). There are sesame, poppy-seed, onion, garlic, and everything bagels. In how many ways can I fill my order of 13 bagels? I expect an answer as a binomial coefficient, then set up in terms of multiplication and division so that I can see that you know how to compute the binomial coefficient. The final calculator answer is not needed.

   A picture with bagels and dividing lines might help but is not required.

6. We say that an integer b is divisible by an integer a (written $a|b$) just in case there is an integer k such that $ak = b$.

Say what the statement "the divisibility relation is transitive" means (the point is to demonstrate that you know what the word "transitive" means) and prove it, using nothing more than basic algebra. Be sure to use proof strategy...

7. Do one of the two induction proofs.

    (a) Prove by mathematical induction that the sum

$$\sum_{i=0}^{n} 3^n = 1 + 3 + 9 + \ldots + 3^n$$

is equal to $\frac{3^{n+1}-1}{2}$, for each natural number $n \geq 0$.

Be sure to clearly identify the basis step and the induction step. In the induction step, identify the inductive hypothesis and the goal to be proved, and indicate where in your proof the inductive hypothesis is used. A significant amount of credit will be based on setting up the proof correctly.

(b) Prove by mathematical induction that $10^n - 1$ is divisible by 3 for each natural number $n \geq 0$.

Be sure to clearly identify the basis step and the induction step. In the induction step, identify the inductive hypothesis and the goal to be proved, and indicate where in your proof the inductive hypothesis is used. A significant amount of credit will be based on setting up the proof correctly.

8. One of the following graphs is planar: draw a planar picture of that one (with vertices properly labelled). One is not planar; use an appropriate formula from the book to demonstrate that it is not planar.

9. Chinese Remainder Theorem (also Test IV, question 2 makeup): Find the smallest positive solution to the system of equations

$$x \equiv 10 \bmod 17$$
$$x \equiv 2 \bmod 23$$

10. Do one of the two parts. If you do both parts your best work will count.

    (a) Show the steps in sorting 3,2,7,6,5 by bubble sort and heap sort. In the heap sort algorithm, you can draw a tree for the part that is a heap or being made a heap and write the sorted part of the list separately.

(b) Prove that in any finite graph, there must be two vertices with the same degree.