# Math 414 Test Review, Fall 2016

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Here I review what you are expected to be able to do on Test II on Wednesday the 19th. The preliminary version of the theorem list to be attached to the exam will appear here.

The format of the test will be the same as that of the last one: four sets of two or three questions, of which you need to do two questions per set, with the one you do better on counting considerably more.

This is **finished** though further revisions are not impossible. I have prepared lists of problems for review and indicated which theorems will be listed, though I haven't given the text for all of them: if I get the text of the theorem and definition handout ready, it will be posted as well.

I will have office hours roughly from 9:15 to 5 on Tuesday (barring family emergency, which is **not** impossible: my father-in-law is in the hospital.) If you look at one of the problems I supply for review and have no idea how to do it...that is a signal that you need to ask me for help, by email or in person. The problems are not all models of test questions (there are far too many of them) but they are in my mind things which would help you form the right habits to answer test questions I might come up with.

A review of content by section:

**chapter 5:** This was covered on the last test, but of course the content of the course is cumulative. Some theorems from chapter 5 will be cited in the theorem list.

I'll supply theorem 5.3A and theorem 5.3B for reference. Other theorems in chapter 5 you should know. There might be some application of theorem 5.4, and if there is I will provide the statement of this theorem.

**6.1:** I might ask you again to write a proof of the Nested Interval Theorem. The statement of the theorem will be given in this case.

**6.2, 6.3:** You do need to know what a cluster point is. The Bolzano Weierstrass theorem is important in being a principal tool for showing that cluster points must exist. The statement of the theorem will be provided in the formula sheet. You need to know how to prove this theorem using the bisection method, and in general you should be able to prove some results using the bisection method. I will be happy to review this in class.

Theorem 6.2 and theorem 6.3 will be stated (since you will have proving theorem 6.3 as an option).

**6.4:** You should know what a Cauchy sequence is. You should have some idea how to prove that a sequence is Cauchy (exercise: prove that the sequence of partial sums of a geometric series with $|r| < 1$ is Cauchy *without* appealing to the fact that it converges).

I might or might not state the definition of a Cauchy sequence: you should know it, and you should know the basic theorem about them (a sequence has a limit iff it is Cauchy).

**6.5:** The completeness property for sets (the least upper bound property). Be able to prove it by the bisection method. Understand the definition of least upper bounds and greatest lower bounds and be able to do some proofs using these concepts. Exercise 6.5.4 is very likely to be an examination question.

I will not necessarily provide definitions of least upper bounds, greatest lower bounds, or the statement of the Completeness Property for sets, as this is content with which you should be familiar.

**chapter 6 problems:** 6.2.2, 6.4.1, 6.4.2, 6.5.1, 6.5.4, problem 6–2

**chapter 7:** The basic tests for convergence in this section will appear on your theorem list. It is important to remember to cite them in proofs when you need to (for example, I expect sadly to be marking off for people failing to mention the tail convergence theorem when they need to).

Theorems 7.2A, 7.2B, 7.2C, 7.2D, 7.3, 7.4A, 7.4B, 7.5B will be stated. As noted, some of these you may be asked to prove.

**7.1:** Basics of what a series are versus a sequence should be review for everyone. The fact that any sequence can be converted to a series by using the telescoping series trick might get some play.

**7.2:** You should be able to prove any of the four theorems in this section. In proving them, you will be allowed in certain cases to use corresponding theorems about sequences, but you need to actually explain how they are used: be fluent in talking about the sequence of partial sums of a sequence and doing calculations with these partial sums. **Read** the proofs the author gives in the section for these theorems. Practice by closing your book and trying to write your own proofs, then comparing.

**7.3:** Understand the definitions of absolute and conditional convergence. Be ready to prove the theorem that a series which converges absolutely therefore converges.

**7.4:** Be able to apply the ratio test (notably to power series). Be able to prove the ratio test as a theorem. The proof of the ratio test **will** be an exam question: be ready to write it. Be able to apply the root test; I will not ask you to prove it and very likely will not ask you to use it, but it might prove usable on a problem where I do not intend it to be used, so there it is.

**7.5:** I am not likely to do an application of the integral test: it is not impossible that I might ask you to prove it. I am much more likely to present you with an opportunity to use the asymptotic comparison method than to ask you to prove it.

**7.6:** The statement of the alternating series test will appear. You will see some applications of it; you may have the opportunity to prove it, but it will not be unavoidable. You should know the error bound for partial sums of the alternating series test which I will probably **not** state: the error in a partial sum as an approximation to the limit is less than or equal to the absolute value of the next term in the series.

**7.7:** I am not going to ask you to prove anything about rearranging order of series. You should be aware of the fact that order of an absolutely convergent series can freely be rearranged without changing its value, and that this is not true for conditionally convergent sequences. If I come up with a nice example I might possibly present the same series in two different orders and ask you to show that the values are different – you should then be able to point out why this is possible.

**chapter 7 problems:** 7.1.1, 7.2.1, 7.3.1, 7.3.3, problem 7–3

**chapter 8:** The proof of theorem 8.1 (existence of radius of convergence) is likely to be on the test (and so of course it will be stated), as are problems on computation of the radius of convergence of given series. On specific series, you might need to explain convergence behavior at the endpoints; I will not ask about Abel summation. You should know the facts about addition and multiplication of power series. The linearity theorem for power series is fair game: I consider the multiplication theorem to be too involved for a test question, though this is less true if I restrict myself to the multiplication theorem for non-negative series.

I might ask you to compute the first few terms of the product of two power series, just to test understanding of how we multiply them.

Theorem 8.1 will be stated; theorem 8.3 will be stated and you may be called upon to prove it; theorem 8.4 will not be stated and you will not be asked to prove it but you **will** be expected to be able to compute the first few terms of the product of two power series. It is remotely possible that I might give you the option of proving the theorem on multiplication of series with positive terms, in which case of course it will be stated.

**chapter 8 problems:** 8.1.1, 8.3.1, problem 8–2.

**chapters 9 and 10:** I regard these as largely review sections. Look at the homework questions I asked to see what sorts of things I might ask. The terminology in section 10 is not all a matter of review: I may test you explicitly on understanding of certain notations (which will mean that certain definitions will not appear on the definition and theorem sheet on the test: you will be explicitly warned where this is the case.

I do not at this time expect to supply any definitions from chapter 9 or 10. In chapter 9 this is largely a matter of review: in chapter 10 there are definitions which I expect you to be able to use which I will not supply, so be sure to read them carefully. I may directly test your ability to explain what his notation "for all $x \approx a$, $P(x)$" means.

**chapter 9 and 10 problems:** 9.2.3, 9.3.1. 10.1 parts 1 to 3, 10.2.1, 10.2.2, 10.3.4 .

**chapter 11:** This will be on the exam. I intend to cover section 11.4 and 11.5 on Friday which means they will be eligible for test coverage.

Be familiar with the definitions of continuity and limit of a function in his forms (though use of the usual Calc II forms will not penalize you: the usual form of the definition of limit appears as a theorem). Notice that he does not define continuity in terms of limits initially. You do need to understand infinite limits and limits at infinity.

Be ready to write an $\epsilon - \delta$ limit proof of the usual kind as an exam question.

Be able to prove the linearity theorem for continuity or for limits.

You need to be familiar with his definitions of continuity and limits (or with the usual ones: good familiarity with the usual forms of the definitions will also suffice to prove theorems). I will not supply definitions of these concepts.

I am not going to supply basic limit theorems familiar to you from Calc I. I will state theorems 11.3C, 11.3D. Theorem 11.4B I might ask you to prove. I should not need to state theorem 11.54A.

I stopped on Friday at theorem 11.5: nothing after theorem 11.5 will be on the test. Theorem 11.4D I advised you to study: you **know** this theorem in the sense of being able to apply it; if I ask you to prove it I'll state it.

**chapter 11 problems:** 11.1.1, 11.1.4 (using the given fact and otherwise just the definition), 11.3.4, 11.3.5, 11.4.2 (this is a matter of looking up the right theorems), 11.5.4, problem 11–1, 11–3 (these "problems" are quite hard, but good to think about; not really test question models, but useful for getting concepts straight).