

Math 175, section 5: Test I

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This exam begins at 5 pm and ends at 6:50 pm.

Calculators with graphing or symbolic computation capability are not allowed. You should not use a calculator on this exam except to do addition, subtraction, or multiplication of whole numbers (or division if there is no remainder). *Exact* answers are required in all problems, not calculator approximations (you may give a calculator estimate in addition to an exact answer – this would be appropriate in the work problem – but not in place of it). Also, any calculation you do with your calculator should be set up on your paper so I can see what you did.

1. A triangle is shown. Set up the area of this triangle as an integral or sum of integrals in two different ways, integrating with respect to x and then integrating with respect to y . Evaluate one of the integrals or sums of integrals.

2. Determine the area of the region bounded by $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$, $y = \sin(x)$ and $y = \cos(x)$.

3. For what value of x in the interval $[1, 4]$ is the value of the function $f(x) = x^2$ equal to the average value of the function on that interval?

4. Sketch the region bounded by the curve $y = 4 - x^2$ and the x -axis and sketch a typical “slice” of the solid obtained by revolving this region around the x -axis. Set up and evaluate an integral which will give the volume of this solid.

Sketch a typical “slice” of the solid obtained by revolving this region around the line $y = -1$. Set up but do not evaluate an integral which will give the volume of this second solid.

Finally, set up but do not evaluate an integral, obtained by the method of disks and washers (not cylindrical shells!) which will give the volume of the solid obtained by revolving this region around the y -axis. (Think!)

5. Sketch the region bounded by the curve $5x - x^2$ and the x -axis.

Determine the volume of the solid obtained by rotating this region around the y -axis by the method of cylindrical shells: sketch a typical cylindrical shell, indicating its radius and height (you can do this on your sketch of the region), then set up and evaluate the integral which gives the volume.

Set up but do not evaluate the integral which gives the volume of the solid obtained by rotating the same region around the line $x = -3$.

6. Sketch the region bounded by the lines $x = 2$, $x = 4$, the line $y = 10 - 2x$, and the x -axis.

We consider the solid obtained by revolving this region around the y -axis.

Set up the integral or sum of integrals obtained by the method of disks and washers which evaluates to the volume of this solid. You do not need to evaluate it.

Set up the integral or sum of integrals obtained by the method of cylindrical shells which evaluates to the volume of this solid. You do not need to evaluate it.

7. Do one of the two problems. If you do both, the best one will be graded, and good performance on both will be good for extra credit.

- (a) A pit in the ground takes the form of an inverted square pyramid, 100 feet deep in the center and 500 feet on a side at ground level. The pit is entirely filled with sand, which for purposes of this problem weighs 150 pounds per cubic foot.

The bed of one of our monster sand trucks is ten feet above the ground. Set up and evaluate an integral which determines the amount of work in foot-pounds required to empty all the sand in the pit into trucks.

- (b) A rope 100 feet long is suspended from the top of a sufficiently tall building. The rope weighs one pound per foot. At the end of the rope is a 5 pound bucket which starts with 50 pounds of sand in it, which is leaking: it leaks a pound of sand for each foot it is drawn upward. How much work is done (on bucket, rope, and sand) in hauling the bucket to the top of the building?