

# Math 175, section 5: Final Exam

M. Randall Holmes

May 7, 2001

This exam begins at 5:40 pm and ends at 7:40 pm.

You are permitted any books or notes you bring plus any calculator you care to use. You may not consult your neighbors.

Note that in some problems you have a choice of which parts to do. Read the directions carefully. ***You should do sufficient parts of each multi-part problem for full credit before working on any additional parts.***

In addition, the lowest numbered problem other than problems 3 or 6 (which actually count as two problems each) will be dropped.

1. chapter 6: Determine the area of the region in the first quadrant bounded by the graphs of  $y = x^3$  and  $y = x^{\frac{1}{3}}$ . Show all work. Give an exact answer.

2. chapter 6: volumes.

Determine the volume of the solid obtained by revolving the pictured region around the  $y$ -axis, first by the method of disks and washers, then by the method of cylindrical shells.

The pictured region is bounded by the  $x$ -axis, the line  $x = 1$  and the line  $y = 4 - 2x$ .

Show complete work and give exact answers.

3. Techniques of integration: do two of the following three problems (counting as two full problems on this exam). If you do all three, the best two will count. In all parts of this problem, you must find integrals by hand, by the specified methods, and show all supporting work. Symbolic capabilities of calculators can be used only as a check.

(a) chapter 7: Integration by parts; show detailed work.

$$\int x^2 \sin(2x) dx$$

(b) chapter 7: Trig substitution; show detailed work.

$$\int \frac{1}{\sqrt{16+x^2}} dx$$

Supplying the answer from an integral table carries no credit at all.

(c) chapter 7: Partial fractions decomposition; show detailed work.

$$\int \frac{x - 1}{x^2 + 5x + 6} dx$$

4. chapter 7: numerical integration problem

Approximate  $\ln 4$  to one decimal place of accuracy (error less than .05) by computing an estimate of

$$\int_1^4 \frac{1}{x} dx$$

using the trapezoid rule. Be sure to justify your error bound using the appropriate formula. You may justify your bound on the second derivative using a graph.

You are expected to determine the correct number of partitions using the error bound formula (found in your book), then actually compute the estimate using the trapezoid rule.

5. chapter 9: choice between an Euler's method problem and a separable differential equation. If both are done, the best will count.

(a) Euler's method example

Use Euler's method with four steps to estimate the value of  $y(2)$  where  $y$  is the solution of the initial value problem  $y' = y; y(0) = 1$ . Record values to at least four decimal places.

Note that you should be able to determine the exact solution quite easily for comparison!

(b) separable initial value problem

Determine the solution of the initial value problem

$$y' = xy; y(0) = 2.$$

6. Chapter 11 (and 10) problems: do two of the four problems given. This counts as two full problems on the exam. If more are done, the best two will be counted.
- (a) chapter 11: convert the repeating decimal  $0.0242424\dots$  to a geometric series and evaluate it using the formula for evaluation of a geometric series, showing all work.

(b) chapter 11: apply the Integral Test to the series

$$\sum_{n=1}^{\infty} \frac{1}{n^4}$$

to determine whether it converges. This will require evaluation of an improper integral, which should be given in complete detail.

Make sure that you state all necessary conditions on the Integral Test.

Do not just say “it’s a  $p$ -series”; no credit will be given for this answer.

- (c) Chapter 11: Apply the Comparison or Limit Comparison Test to determine whether the series

$$\sum_{n=1}^{\infty} \frac{n+1}{n^3+1}$$

converges. Justify the inequalities you state.

- (d) Chapter 10: Determine which of the two pictured circles is represented by the polar equation  $r = 4 \sin(\theta)$ . Present calculations verifying the equivalence of the polar equation to the cartesian equation for the given circle.

7. Power series. Do both parts.

- (a) Determine the MacLaurin series for the function  $f(x) = (1+x)^{-3}$ . Give the first four nonzero terms for full credit; correct sigma notation is good for a small amount of extra credit.
- (b) Identify the function represented by the power series  $\sum_{n=0}^{\infty} (-4)^n x^{2n+1} = x - 4x^3 + 16x^5 - 64x^7 \dots$  (Hint: it is a geometric series). Determine the radius and interval of convergence of this series.