Write power series for the following functions. All of these come from the series for $\frac{1}{1-x}$, differentiation, and integration.

1. $\frac{1}{3+x}$ (series in powers of $x$) What is the interval on which this series converges?

2. $\frac{1}{3+x}$ (series in powers of $x + 2$) What is the interval on which this series converges?

3. $\frac{1}{1+x^3}$

4. $\frac{x}{1+x^2}$

5. $\ln(1+x)$ (hint: you find this by integration).

6. $\frac{1}{(3+x)^2}$ (find this by differentiating one of the first two series).

For each of the following series, write down the sum of the first four or five terms followed by dots... This is practice in reading the notation.

Determine the interval of convergence of each of the series (use the ratio test or root test to find the radius of convergence, from this determine the interval, and check the endpoints separately). The interval might be the whole real line.

1. $$\sum_{n=0}^{\infty} \frac{1}{n!}$$
2. \[ \sum_{n=0}^{\infty} \frac{(x - 2)^n}{3^n} \]

3. \[ \sum_{n=0}^{\infty} 10^n x^n \]

4. \[ \sum_{n=0}^{\infty} \frac{x^{2n}}{4^n} \]

(be careful)

5. \[ \sum_{n=0}^{\infty} \frac{x^n}{n^2} \]

6. \[ \sum_{n=0}^{\infty} n! x^n \]