Math 175 Test III

Dr. Holmes

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This exam will last from 9:40 to 10:35 am. You may use a plain scientific
calculator with no graphing or symbolic computation capabilities. Please
write your name on your exam paper and record the number on the inside
front cover of your exam paper by which your exam grade will be posted
on the web. Please write your name on any blue book you use and write
all work you do in the blue book(s), preferably in order of problem number
(though I will look around if I have to!) You will return your exam paper
and all blue books, used or unused, to me at the end of the exam.

1. Geometric series

   (a) Compute

   \[ \sum_{n=1}^{\infty} \left( \frac{2}{3^n} + \frac{3}{2^n} \right) \]

   (b) For what values of \( x \) does \( \sum_{n=0}^{\infty} \frac{x}{(x-1)^n} \) converge? When it does
       converge, what does it converge to?

2. Integration by parts: do the indicated integrations by parts, showing
   all work (tell me what \( u, v, du, dv \) are, etc.)

   (a) \[ \int \ln(x) \, dx \]

   (b) \[ \int x^2 e^{3x} \, dx \]
3. Integration by partial fractions: be sure to show all work.
   Evaluate
   \[ \int \frac{2x - 1}{x^2 - 4} \, dx \]

4. Improper integrals
   Evaluate
   \[ \int_{0}^{1} \frac{1}{x^{1/3}} \, dx \]
   Be sure to write it as a limit. Show all work.

5. The Integral Test
   Determine whether
   \[ \sum_{n=1}^{\infty} \frac{1}{n(n + 1)} \]
   is convergent using the Integral Test. State the conditions on the relevant function which have to hold for the Integral Test to apply (you do not need to verify them, just say what they are). Set up and evaluate the appropriate improper integral and state your conclusions about the series.

6. The comparison tests
   Determine whether
   \[ \sum_{n=1}^{\infty} \frac{1}{n^2 - e^{-n}} \]
   is convergent using a comparison test. You may use known facts about convergence of series. You need to state relevant inequalities and convergence facts.