This exam begins at 940 am and ends at 1035 am. You may use a standard scientific calculator without graphing or symbolic computation capabilities; you may not use any other calculator. Cell phones must be turned off and out of sight.

Two blue books are enclosed. Write your name on your exam paper and record the magic number inside the front cover by which your grade will be posted. Write your name on any blue book that you use. It will be appreciated if problems appear in order on your test paper. Do not write any answers on your exam paper. Return the exam paper and all blue books (including blank ones).

You may drop one question; if you do all questions, your best work will count.
1. Set up the integral representing the surface area of the solid of revolution obtained by revolving \( y = x^3, 0 \leq x \leq 2 \) around the \( x \)-axis. For some extra credit, you may evaluate the integral (it can be evaluated).

2. Solve the separable differential equation

\[
\frac{dy}{dx} = \frac{x}{y^2}.
\]

Give your solution in the form \( y = f(x) \) (solve for \( y \) as a function of \( x \), don’t just leave it in implicit form as the book so often does). You need to find all solutions (don’t forget the constant of integration!)

3. The half-life of Ytterbium-91 is 237 years (I just made up this isotope and this half-life, don’t look it up in the CRC!). A pure preparation of Ytterbium-91 was left on a shelf at Hanford, and when it was found 7 percent of it had decayed. How long was it left on the shelf? Hint: how much of it was left?

4. A bucket weighing 35 pounds is hanging off a building at the end of a rope 100 feet long which weighs .125 pounds per foot. How much work will be done on the rope and the bucket if it is pulled to the top of the building? Set up the integral and compute the numerical solution.

5. A conical pit 20 feet deep with radius 10 feet at the top is full of water. The water has to be pumped into a tank on the back of a truckbed: the hose into the tank is 15 feet above the edge of the pit. How much work is done on the water if all of it is pumped into the tank? Water weighs 62.4 lb/ft\(^3\) In this problem it is sufficient to set up the integral.

6. Compute the center of mass of a plate of uniform density 1 covering the region bounded by the coordinate axes and the line \( y = 1 - \frac{x}{2} \) by computing the mass of the plate, the moment of the plate around the \( x \)-axis and the moment of the plate around the \( y \)-axis (clearly identifying each) then computing the \( x \) and \( y \) coordinates of the center of mass.
7. Sequences. If you have to compute a limit, show supporting work. If you use L’Hôpital’s rule, be sure to say so and give some indication of why the rule applies.

(a) Write a closed form formula (not a recursive formula) for the sequence $a_1 = 2; a_2 = \frac{2}{3}; a_3 = \frac{2}{5}; a_4 = \frac{2}{7}$, etc.

(b) Write the next three terms of the sequence defined by $a_1 = 1; a_2 = 3; a_{n+2} = a_n + a_{n+1}$ (this is the sequence of Lucas numbers).

(c) Determine $\lim_{n \to \infty} \frac{2n-1}{3n+1}$. Show work supporting your answer.

(d) Determine $\lim_{n \to \infty} \frac{2}{e^n}$. Show work supporting your answer.