This exam will begin at 9 am and end officially at 10:15 am (the usual class period). What will actually happen at 10:15 is that I will give a five minute warning.

You may use your writing instrument, your test paper, and a non-graphing calculator (which you probably have no particular use for on this exam); nothing else.

Cell phones must be turned off and out of sight.
1. Compute the antiderivatives using the method of substitution. Please show all details of your substitutions.

(a) 
\[ \int \frac{x}{x^2 + 1} \, dx \]

(b) 
\[ \int x\sqrt{x + 3} \, dx \]
2. Compute the antiderivative.

\[ \int \frac{1}{1 + 9x^2} \, dx \]
3. Compute the antiderivatives. Show detailed supporting work.

(a) \[ \int \sin^2(x) \, dx \]

(b) \[ \int \sin^3(x) \cos^2(x) \, dx \]
4. Determine the area of the region bounded by $y = x + 2$ and $y = 4 - x^2$ by setting up and evaluating an appropriate definite integral. A sketch is provided.
5. Do one of the two parts. If you do both, your best work will count. If you do very well on both, you might get additional credit.

(a) The base of a solid is the region bounded by $y = 1 - x^2$ and the $x$-axis. Its cross-sections parallel to the $y$ axis are squares. Compute its volume by setting up and evaluating an appropriate definite integral.

(b) Determine the average value of $f(x) = x^3$ on the interval $[1, 3]$. Find a number $c$ in the interval $[1, 3]$ such that $f(c)$ is equal to this average value.
6. Determine the volume of the solid obtained by revolving the region bounded by \( y = x^2 - 2x \) and the \( x \) axis (sketch provided) around the \( x \)-axis, by setting up and evaluating an appropriate definite integral.

Set up but do not evaluate the integral you would need to evaluate to determine the volume of the solid obtained by revolving the same region around the line \( y = -3 \).
7. Set up the definite integrals representing the volume of the region obtained by revolving the region bounded by $y = x^3$, $y = 8$ and the $y$-axis (sketch given) around the $y$-axis, first by the method of disks and washers (6.3) then by the method of cylindrical shells (6.4).

Evaluate one of them (of course if you evaluate both you have a check on your work; if you evaluate both I will give credit for the better of the two evaluations).
8. Find the antiderivative

\[ \int \sqrt{4 - x^2} \, dx \]

using the substitution \( x = 2 \sin(\theta) \). A relevant right triangle is shown.