Sample Test Questions for Test III

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Keep an eye on this document. I may add things to it. This should be representative of the kinds of questions you will see on Test III – it may be longer than your Test III paper. There is no guarantee that every kind of question appearing here will appear on Test III, nor that every question on Test III should have a close model here. But if you are prepared to write this paper, you should be prepared for the test.

You may hand this paper in on Friday to be returned graded on Monday, or on Monday to be returned graded Wednesday. This is optional but strongly recommended.

I recommend writing answers on separate paper and giving yourself plenty of room to write each answer.

PLEASE bring typos to my attention as soon as possible. Use email! Solutions will be posted on Wednesday.
1. Use the Comparison Test or the Limit Comparison Test to determine for each of the following series whether it converges or diverges.

You must state which series you are comparing it to (this will be a geometric series or \( p \)-series) and briefly indicate why the series you are comparing to converges or diverges.

You must then indicate the relation of the terms of the series you are working on to the terms of the series you are comparing it to which enables you to conclude that the series you are working on converges or diverges (write out an explicit inequality or an explicit limit statement about ratios). The words you use must indicate understanding of the test you are using.

(a) \[ \sum_{n=0}^{\infty} \frac{3}{4^n + 1} \]

(b) \[ \sum_{p=2}^{\infty} \frac{1}{\sqrt{p} - 1} \]

(c) \[ \sum_{n=1}^{\infty} \frac{1}{2n^3 - 1} \]

(d) \[ \sum_{n=1}^{\infty} \frac{n^2}{3^n} \]

(hint: compare with \[ \sum_{n=1}^{\infty} \frac{1}{2^n} \]

using the LCT; use the clauses which mention zero or infinite limits)
2. Determine whether the following series with alternating signs converge: state for each series the conditions needed for the Alternating Series test to apply (this does not mean copy the statement of the theorem, this means state the actual inequalities and limit statements for the series you are looking at) and say whether it converges or not. Then explain why the series converges absolutely, or why it does not converge absolutely. You might have to mention other tests in answering this last part.

(a) \[ \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{\sqrt{n}} \]

(b) \[ \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^3 + 1} \]

(c) \[ \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n + 1}{n} \]
3. Each of the following sequences converges by the alternating series test. State the facts that must be stated to see that the test applies, as in the previous problem.

Tell me how many terms of each series need to be added to give an estimate within .0001 of the value of the series. Explain why.

Then compute the actual estimate of one of them, showing the actual setup of adding all required terms of the series on your paper. You should be able to see that you do not want to do this calculation for the other series!

(a) \[ \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \]

(b) \[ \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n2^n} \]
4. Use the Ratio or Root Test to determine the convergence or divergence of each of the following series. If the Ratio or Root Test is inconclusive, be sure to explain why it is inconclusive, and give an answer to the question as to whether the series converges or diverges using a different test.

(a) \[ \sum_{n=0}^{\infty} \frac{(n + 1)}{n!} \]

(b) \[ \sum_{n=0}^{\infty} \frac{1}{n^4} \]

(c) \[ \sum_{n=0}^{\infty} \left( \frac{1}{\arctan(n)} \right)^n \]
   (hint: this uses a limit fact about arctan that you ought to know, along with a Test)

(d) \[ \sum_{n=0}^{\infty} \frac{n!}{n^{100}} \]

(e) \[ \sum_{n=0}^{\infty} \frac{n^2}{3^n} \]
5. Write each of the following series as a function of $x$ and indicate the domain on which the series converges. Hint: what kind of series are these?

(a) \[ 1 + x + x^2 + x^3 + \ldots \]

(b) \[ 1 - 3x + 9x^2 - 27x^3 + \ldots \]

(c) \[ 1 - x^2 + x^4 - x^6 + x^8 - \ldots \]
6. Develop a power series for $\arctan(x)$ by exhibiting a series for $\frac{1}{1+x^2}$ then integrating it term by term.

Use the series to evaluate $\arctan\left(\frac{1}{2}\right)$ with an error of less than .01 (you need to justify how many terms you add and set up the sum on your paper; I know that your calculator can compute arctangents).
7. Compute the radius and interval of convergence of each power series using the Ratio Test. Remember that if the radius of convergence is finite you need to test the endpoints.

(a)  \[ \sum_{n=0}^{\infty} \frac{x^n}{3^n} \]

(b)  \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n} \]

(c)  \[ \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} \]

(d)  \[ \sum_{n=1}^{\infty} n^n x^n \]

(e)  \[ \sum_{n=1}^{\infty} \frac{x^n}{n^3} \]