This is longer than an actual test. Reduction formulas which you need will be supplied; but be aware you might have to prove one. You will not be supplied with the basic formulas for 7.3; you might get some hints though.
1. 10.7 compute some Taylor series

(a) Compute the first four terms of the Taylors series for $f(x) = \frac{1}{x^2}$ centered at 1 from the official formulas. Show all details (computations of all needed repeated derivatives, computation of the specific values of these derivatives that are needed, show that you know how to get the coefficients for the series from these values.

(b) Compute the first four nonzero terms of the MacLaurin series for $\sin(x^2)$ using the series for $\sin(x)$ which you should know.

(c) Compute the first four nonzero terms of the MacLaurin series for $x^2 e^{2x}$ (again, you should be able to compute this without actually taking repeated derivatives of this function).
2. 10.7 estimate a bad integral using a series

Compute \( \int_{0}^{1} \sin(x^2) \, dx \) within .001 using the Maclaurin series for an antiderivative of \( \sin(x^2) \), which you can compute by integrating the series for \( \sin(x^2) \) term by term. You can check the error bound by using the fact that the series alternates. I know that your calculator can very likely compute this value: I need you to show me the series you use, the actual addition you carry out, and the reason that you know that the error is less than .001 (which cannot involve an appeal to your calculator answer).
3. 7.2 prove cosine reduction formula

Prove the reduction formula for cosine

\[ \int \cos^n(x) \, dx = \frac{1}{n} \cos^{n-1}(x) \sin(x) \, dx + \frac{n-1}{n} \int \cos^{n-2}(x) \, dx \]

Hint: use integration by parts.
4. 7.2 sine and cosine examples. Please simplify your answers algebraically.

(a) \[ \int \cos^4(x) \, dx \]

(b) \[ \int \cos^2(x) \sin^2(x) \, dx \]

(c) \[ \int \cos^3(x) \sin^4(x) \, dx \]
5. 7.2 secant and tangent examples

(a) \[ \int \tan^3(x) \, dx \]

(b) \[ \int \sec^3(x) \, dx \]

(c) \[ \int \sec^2(x) \tan^2(x) \, dx \]
6. 7.3 Compute

$$\int \sqrt{4 - x^2} \, dx$$

using a trigonometric substitution.
7. 7.3 Compute

\[ \int \frac{x}{\sqrt{x^2 + 1}} \, dx \]

using a trigonometric substitution. Compute it by a different substitution and check that you get the same answer.
8. Compute an example directly

Estimate \[ \int_0^2 x^3 \, dx \]

using the Trapezoid Rule with four partitions. Show your complete calculation.

Before computing the estimate or the exact answer, draw a picture of the region and explain using your picture (and a few words) whether you expect your estimate to be too large or too small.

Compute an upper bound on the error in this estimate using the Trapezoidal Rule error bound formula (these formulas will be given on the test, but you can look it up now).
9. 7.8 compute number of partitions needed for an estimate.

Determine how many partitions you need to compute

$$\ln(3) = \int_{1}^{3} \frac{1}{x} \, dx$$

with an error of less than .01 using Simpson’s Rule. Compute the estimate. You must show all details of your calculation; I know that your calculator can handle logarithms.