# Study Guide for Test IV, Math 301

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In gneeral, this exam will be mostly computationally oriented. The one kind of extended "proof" you may expect is algebraic computation with the dot product. You really should have a TI-89-equivalent calculator at this exam: if you do not have one you will be at a disadvantage.

There may be brief theoretical questions (parts of larger numbered questions) testing understanding of specific facts mentioned in this outline.

The number of sections here is greater than the likely number of problems (though not as much as on the last exam!) so you may expect that some sections may be unified into a single problem or even omitted altogether.

I have provided sample problems mostly as models for conceptual questions that I might ask. The homework problems (especially the ones I chose to grade) should give adequate models for the computational questions I am likely to ask.

**section 5.1:** Know what an eigenvector and an eigenvalue are. Given an eigenvector, be able to determine the corresponding eigenvalue (this is trivial). Given an eigenvalue, be able to determine the eigenspace (the set of all eigenvectors for that eigenvalue.

Be aware that eigenvectors for distinct eigenvalues are linearly independent.

You should know what to do with problems like 31, 32.

**section 5.2:** Be able to set up the characteristic equation for a 2 by 2 or 3 by 3 matrix. Be able to solve the characteristic equation to find eigenvalues, and then proceed to find eigenvectors for each eigenvalue.

Be aware that a matrix has zero as an eigenvalue exactly if it is singular (has zero determinant, does not have an inverse).

Know the definition of similar matrices, and be aware that similar matrices have the same eigenvalues (but not the same eigenvectors). Also be aware that row operations do not preserve either eigenvectors or eigenvalues.

**section 5.3:** Be able to diagonalize matrices. Be aware that it is easy to diagonalize a matrix if you are given a basis of eigenvectors for that matrix (I might give you a basis of eigenvectors in an actual problem – if I don't give you the corresponding eigenvalues, remember that these are very easy to find).

Know why it is easy to compute large powers of matrices given in diagonalized form.

Be able to recognize a matrix which is not diagonalizable and explain why it isn't (it won't have as many distinct eigenvalues as columns).

**section 5.4:** Be able to find B-matrices representing linear transformations from a vector space V to itself, where B is a basis for V. But I am quite likely not to ask about this section at all.

**section 5.5:** Be able to find complex eigenvalues and the corresponding eigenvectors. Be able to carry out the kind of factorization described in theorem 9, p. 340.

**section 5.8:** Be able to estimate eigenvectors by both the power method and the inverse power method.

**section 6.1:** Know how to compute dot products, and how to compute lengths of vectors, distances between vectors, and determine whether two vectors are orthogonal using the dot product. I will not ask about general angles between vectors, but do be aware that orthogonality has a geometrical meaning: orthogonal vectors are at right angles to one another.

Be familiar with the algebraic properties of the dot product. Be able to do problems like problem 20, part d.

Know what the orthogonal complement $W^{\perp}$ of a subspace $W$ is (p. 380).

**section 6.2:** Know the definition of an orthogonal set. Know that an orthogonal set is linearly independent, so the only requirement for an orthogonal set to be a basis is that there are enough vectors in it.

Be able to compute orthogonal projections and understand their geometrical meaning.

Given vectors $a$ and $b$, be able to express $a$ as the sum of a scalar multiple of $b$ and a vector orthogonal to $b$. Be aware that the length of the vector orthogonal to $b$ is the distance from the point represented by $a$ to the line through the origin in the direction of $b$, and that the point represented by the scalar multiple of $b$ is the closest point on the line to the point represented by $a$.

Know what an orthonormal set and basis are.

Know that a matrix $U$ has orthonormal columns just in case $U^TU = I$.

Know that an orthogonal matrix is a square matrix with *orthonormal* columns, or equivalently a square matrix $U$ such that $U^TU = I$.

Understand the proof that an orthogonal basis has orthonormal *rows* (problem 28). If I am overcome by the urge to put on a proof, this is a likely one (for one thing, we have discussed it at least three times in class!!!)

You should be able to prove using algebraic properties of the dot product and the formula for orthogonal projections that $a - \mathtt{proj}_b a$ is orthogonal to $b$.

**section 6.3:** Be able to compute orthogonal projections onto subspaces with no more than 2 dimensions.

Be able to apply the Best Approximation theorem to determine the distance between a point and a subspace.

**section 6.4:** Be able to carry out the Gram-Schmidt process to find an orthogonal basis of a subspace with no more than 3 dimensions (the space in which the subspace lives may have more dimensions, of course).

I will not ask for QR factorizations.

**section 6.5:** Be able to find least squares solutions computationally, and understand what they are. A problem like problem 13 or 14 should not be a mystery.