```
> with(LinearAlgebra);
```

[*Add*, *Adjoint*, *BackwardSubstitute*, *BandMatrix*, *Basis*, *BezoutMatrix*, *BidiagonalForm*, *BilinearForm*, *CharacteristicMatrix*, *CharacteristicPolynomial*, *Column*, *ColumnDimension*, *ColumnOperation*, *ColumnSpace*, *CompanionMatrix*, *ConditionNumber*, *ConstantMatrix*, *ConstantVector*, *CreatePermutation*, *CrossProduct*, *DeleteColumn*, *DeleteRow*, *Determinant*, *DiagonalMatrix*, *Dimension*, *Dimensions*, *DotProduct*, *Eigenvalues*, *Eigenvectors*, *Equal*, *ForwardSubstitute*, *FrobeniusForm*, *GaussianElimination*, *GenerateEquations*, *GenerateMatrix*, *GetResultDataType*, *GetResultShape*, *GivensRotationMatrix*, *GramSchmidt*, *HankelMatrix*, *HermiteForm*, *HermitianTranspose*, *HessenbergForm*, *HilbertMatrix*, *HouseholderMatrix*, *IdentityMatrix*, *IntersectionBasis*, *IsDefinite*, *IsOrthogonal*, *IsSimilar*, *IsUnitary*, *JordanBlockMatrix*, *JordanForm*, *LA_Main*, *LUDecomposition*, *LeastSquares*, *LinearSolve*, *Map*, *Map2*, *MatrixAdd*, *MatrixInverse*, *MatrixMatrixMultiply*, *MatrixNorm*, *MatrixScalarMultiply*, *MatrixVectorMultiply*, *MinimalPolynomial*, *Minor*, *Multiply*, *NoUserValue*, *Norm*, *Normalize*, *NullSpace*, *OuterProductMatrix*, *Permanent*, *Pivot*, *QRDecomposition*, *RandomMatrix*, *RandomVector*, *Rank*, *ReducedRowEchelonForm*, *Row*, *RowDimension*, *RowOperation*, *RowSpace*, *ScalarMatrix*, *ScalarMultiply*, *ScalarVector*, *SchurForm*, *SingularValues*, *SmithForm*, *SubMatrix*, *SubVector*, *SumBasis*, *SylvesterMatrix*, *ToeplitzMatrix*, *Trace*, *Transpose*, *TridiagonalForm*, *UnitVector*, *VandermondeMatrix*, *VectorAdd*, *VectorAngle*, *VectorMatrixMultiply*, *VectorNorm*, *VectorScalarMultiply*, *ZeroMatrix*, *ZeroVector*, *Zip*]

```
> #Problem 1
> Matrix(3,1,[[3],[4],[5]]); #column vector as a 3 by 1 matrix
```

$$\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

```
> Matrix(3,1,[[3],[4],[5]]) + Matrix(2,1,[[-1],[2]]); # you can't
add
> these -- they aren't of the same length. problem 1a

Error, (in rtable/Sum) invalid arguments
```

```
> Matrix(3,1,[[1],[3],[-2]]) - Matrix(3,1,[[-2],[1],[5]]); #problem
> 1b
```

$$\begin{bmatrix} 3 \\ 2 \\ -7 \end{bmatrix}$$

```
> A:=Matrix(2,2,[[2,1],[-1,3]]);
```

$$A := \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$$

```
> B:=Matrix(2,3,[[1,-2,0],[-2,2,1]]);
```

$$B := \begin{bmatrix} 1 & -2 & 0 \\ -2 & 2 & 1 \end{bmatrix}$$

```
> MatrixMatrixMultiply(A,B);  #problem 1b -- it works
```

$$\begin{bmatrix} 0 & -2 & 1 \\ -7 & 8 & 3 \end{bmatrix}$$

```
> MatrixMatrixMultiply(B,A);  # problem 1c -- it doesn't work.

Error, (in LinearAlgebra:-MatrixMatrixMultiply) first matrix column
dimension (3) <> second matrix row dimension (2)

> #Problem 2

> A:=Matrix(3,4,[[1,1,1,4],[1,-2,1,1],[2,3,4,13]]);  #set up the
> augmented matrix for 2a
```

$$A := \begin{bmatrix} 1 & 1 & 1 & 4 \\ 1 & -2 & 1 & 1 \\ 2 & 3 & 4 & 13 \end{bmatrix}$$

```
> A:=RowOperation(A,[2,1],-1);  #add -1 times row 1 to row 2
```

$$A := \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & -3 & 0 & -3 \\ 2 & 3 & 4 & 13 \end{bmatrix}$$

```
> A:=RowOperation(A,[3,1],-2); #add -2 times row 1 to row 3
```

$$A := \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & -3 & 0 & -3 \\ 0 & 1 & 2 & 5 \end{bmatrix}$$

```
> A:=RowOperation(A,[2,3]); #swap rows 2 and 3
```

$$A := \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 2 & 5 \\ 0 & -3 & 0 & -3 \end{bmatrix}$$

```
> A:=RowOperation(A,[3,2],3);  #add 3 times row 2 to row 3
```

$$A := \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 6 & 12 \end{bmatrix}$$

```
> z:=solve(6*z=12,z);  #set z equal to the solution of 6z=12
```

$$z := 2$$

```
> y:=solve(y+2*z=5,y);  #set y equal to the solution of y+2z=5
(Maple
> remembers the value assigned to z)
```

$$y := 1$$

```
> x:=solve(x+y+z=4,x);  #solve for x
```

$$x := 1$$

```
> Matrix(3,1,[[x],[y],[z]]);  #display the solution in vector (column
> matrix) format
```

$$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

```
> x:=evaln(x); y:=evaln(y); z:=evaln(z);  #forget values of these
> variables
```

$$x := x$$
$$y := y$$
$$z := z$$

```
> #problem 2 part b


> A:=Matrix(2,4,[[1,-1,1,4],[2,-1,-1,1]]);
```

$$A := \begin{bmatrix} 1 & -1 & 1 & 4 \\ 2 & -1 & -1 & 1 \end{bmatrix}$$

```
> A:=RowOperation(A,[2,1],-2);
```

$$A := \begin{bmatrix} 1 & -1 & 1 & 4 \\ 0 & 1 & -3 & -7 \end{bmatrix}$$

```
> y:=solve(y-3*z=-7,y);
```

$$y := 3\,z - 7$$

```
> x:=solve(x-y+z=4,x);
```

$$x := 2\,z - 3$$

```
> x:=evaln(x); y:=evaln(y);z:=evaln(z);
```

$$x := x$$
$$y := y$$
$$z := z$$

```
> Matrix(3,1,[[x],[y],[z]]);
```

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

```
> z*Matrix(3,1,[[2],[3],[1]]) + Matrix(3,1,[[-3],[-7],[0]]);
```

$$z \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} -3 \\ -7 \\ 0 \end{bmatrix}$$

```
> #you can also write it this way -- Maple won't compute this written
> this way.


> #Problem 3


> #set up the matrix to compute the inverse
```

```
> A:=Matrix(2,4,[[1,3,1,0],[5,1,0,1]]);
```

$$A := \begin{bmatrix} 1 & 3 & 1 & 0 \\ 5 & 1 & 0 & 1 \end{bmatrix}$$

```
> A:=RowOperation(A,[2,1],-5);
```

$$A := \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & -14 & -5 & 1 \end{bmatrix}$$

```
> A:=RowOperation(A,2,-1/14);
```

$$A := \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & \frac{5}{14} & \frac{-1}{14} \end{bmatrix}$$

```
> A:=RowOperation(A,[1,2],-3);
```

$$A := \begin{bmatrix} 1 & 0 & \frac{-1}{14} & \frac{3}{14} \\ 0 & 1 & \frac{5}{14} & \frac{-1}{14} \end{bmatrix}$$

```
> #Use Maple to compute the inverse directly

> B:=Matrix(2,2,[[1,3],[5,1]]);
```

$$B := \begin{bmatrix} 1 & 3 \\ 5 & 1 \end{bmatrix}$$

```
> C:=MatrixInverse(B);  #notice that we have the same result
```

$$C := \begin{bmatrix} \frac{-1}{14} & \frac{3}{14} \\ \frac{5}{14} & \frac{-1}{14} \end{bmatrix}$$

```
> #Finally, use the matrix inverse to solve the given system

> MatrixMatrixMultiply(C,Matrix(2,1,[[6],[16]]));
```

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

```
> solve({x+3*y=6,5*x+y=16});  #check
```

$$\{y = 1, x = 3\}$$

```
> #Problem 4

> A:=Matrix(3,1,[[1],[-1],[3]]);
```

$$A := \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$$

```
> B:=Matrix(3,1,[[-2],[2],[4]]);
```

$$B := \begin{bmatrix} -2 \\ 2 \\ 4 \end{bmatrix}$$

```
> C:=Matrix(3,1,[[8],[-8],[-6]]);
```

$$C := \begin{bmatrix} 8 \\ -8 \\ -6 \end{bmatrix}$$

```
> #set up the matrix to solve the problem

> E:=Matrix(3,4,[[1,-2,8,0],[-1,2,-8,0],[3,4,-6,0]]);
```

$$E := \begin{bmatrix} 1 & -2 & 8 & 0 \\ -1 & 2 & -8 & 0 \\ 3 & 4 & -6 & 0 \end{bmatrix}$$

```
> E:=RowOperation(E,[2,1],1);
```

$$E := \begin{bmatrix} 1 & -2 & 8 & 0 \\ 0 & 0 & 0 & 0 \\ 3 & 4 & -6 & 0 \end{bmatrix}$$

```
> E:=RowOperation(E,[2,3]);
```

$$E := \begin{bmatrix} 1 & -2 & 8 & 0 \\ 3 & 4 & -6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```
> E:=RowOperation(E,[2,1],-3);
```

$$E := \begin{bmatrix} 1 & -2 & 8 & 0 \\ 0 & 10 & -30 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```
> z:=1;  #choose an arbitrary value for z
```

$$z := 1$$

```
> y:=solve(10*y-30*z=0,y);
```

$$y := 3$$

```
> x:=solve(x-2*y+8*z=0,x);
```

$$x := -2$$

```
> x:=evaln(x); y:=evaln(y); z:=evaln(z);
```

$$x := x$$

$$y := y$$

$$z := z$$

```
> -2*A+3*B+1*C;  #it works...
```

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

```
> #Problem 5
```

```
> det([[1,-2,3],[-1,-1,2],[3,0,-1]]);
```

$$0$$

```
> # since the determinant is 0, the null space is nontrivial.

> A:=Matrix(3,4,[[1,-2,3,0],[-1,-1,2,0],[3,0,-1,0]]);
```

$$A := \begin{bmatrix} 1 & -2 & 3 & 0 \\ -1 & -1 & 2 & 0 \\ 3 & 0 & -1 & 0 \end{bmatrix}$$

```
> A:=RowOperation(A,[2,1],1);
```

$$A := \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & -3 & 5 & 0 \\ 3 & 0 & -1 & 0 \end{bmatrix}$$

```
> A:=RowOperation(A,[3,1],-3);
```

$$A := \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & -3 & 5 & 0 \\ 0 & 6 & -10 & 0 \end{bmatrix}$$

```
> A:=RowOperation(A,[3,2],2);
```

$$A := \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & -3 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```
> y:=solve(-3*y+5*z=0,y);
```

$$y := \frac{5}{3}\,z$$

```
> x:=solve(x-2*y+3*z=0,x);
```

$$x := \frac{1}{3}\,z$$

```
> Matrix(3,1,[[x],[y],[z]]);
```

$$\begin{bmatrix} \frac{1}{3}\,z \\ \frac{5}{3}\,z \\ z \end{bmatrix}$$

```
> #the nullspace is one dimensional and has basis consisting
> #just of (1/3,5/3,1)^T

> #Problem 6

> u:=D(y);
```

$$u := \frac{5}{3}\,\mathrm{D}(z)$$

```
> v:=D(u);
```

$$v := \frac{5}{3}\,(\mathrm{D}^{(2)})(z)$$

```
> D(v)(t)=t*v(t)-2*u(t)+sin(t)*y(t)+cos(t);
```

$$\frac{5}{3}\,(\mathrm{D}^{(3)})(z)(t) = \frac{5}{3}\,t\,(\mathrm{D}^{(2)})(z)(t) - \frac{10}{3}\,\mathrm{D}(z)(t) + \frac{5}{3}\sin(t)\,\mathrm{z}(t) + \cos(t)$$

The system of equations wanted is

x' = u

u' = v

v' = tv -2u +sin(t)y+cos(t)

```
> #Problem 7
```

The equation is

A' = .75 -(7/30)A+(2/20)B

B' = (7/30)A - (7/20)B

Initial conditions are A(0) = 1.5, B(0) = 1

```
> #we can solve this
> A:=evaln(A); B:=evaln(B);
```

$$A := A$$
$$B := B$$

```
> dsolve({D(A)(t) = .75 - (7/30)*A(t) + (2/20)*B(t),
> D(B)(t) = (7/30)*A(t)-(7/20)*B(t),A(0)=3/2,B(0)=1});
```

$$\{\mathrm{A}(t) = \frac{1}{4}\,e^{(1/120\,(-35+\sqrt{385})\,t)}\,(-\frac{1}{11}\,\sqrt{385}-1) + \frac{1}{28}\,e^{(1/120\,(-35+\sqrt{385})\,t)}\,(-\frac{1}{11}\,\sqrt{385}-1)\,\sqrt{385}$$
$$+\frac{1}{4}\,e^{(-1/120\,(35+\sqrt{385})\,t)}\,(\frac{1}{11}\,\sqrt{385}-1) - \frac{1}{28}\,e^{(-1/120\,(35+\sqrt{385})\,t)}\,(\frac{1}{11}\,\sqrt{385}-1)\,\sqrt{385}$$
$$+\frac{9}{2},$$
$$\mathrm{B}(t) = e^{(1/120\,(-35+\sqrt{385})\,t)}\,(-\frac{1}{11}\,\sqrt{385}-1) + e^{(-1/120\,(35+\sqrt{385})\,t)}\,(\frac{1}{11}\,\sqrt{385}-1) + 3\}$$

```
> evalf(%);  #put it into floating point format
```

$$\{\mathrm{B}(t) = -2.783765170\, e^{(-.1281548594\, t)} + .783765170\, e^{(-.4551784739\, t)} + 3.,$$
$$\mathrm{A}(t) = -2.646706180\, e^{(-.1281548594\, t)} + 4.500000000 - .3532938192\, e^{(-.4551784739\, t)}\}$$

```
> #notice that the exponential terms all go to zero, and
> #the limiting quantities of salt in tanks A and B are just
> #fifteen percent of their volume, as one might expect.

> #Problem 8
> x:=evaln(x); y:=evaln(y);
```

$$x := x$$
$$y := y$$

```
> equ1:= D(x)(t)=4*x(t)-3*y(t);
```

$$equ1 := \mathrm{D}(x)(t) = 4\,\mathrm{x}(t) - 3\,\mathrm{y}(t)$$

```
> equ2:=D(y)(t)=2*x(t)-y(t);
```

$$equ2 := \mathrm{D}(y)(t) = 2\,\mathrm{x}(t) - \mathrm{y}(t)$$

```
> x:= t->exp(2*t); y:= t->(2/3)*exp(2*t);
```

$$x := t \to e^{(2\,t)}$$
$$y := t \to \frac{2}{3}\, e^{(2\,t)}$$

```
> equ1;
```

$$2\, e^{(2\,t)} = 2\, e^{(2\,t)}$$

```
> equ2;
```

$$\frac{4}{3}\, e^{(2\,t)} = \frac{4}{3}\, e^{(2\,t)}$$

```
> # it checks for the first set of solutions.

> x:=t->exp(t); y:=t->exp(t);  #output here is a little weird
```

$$x := \exp$$
$$y := \exp$$

```
> equ1;
```

$$e^t = e^t$$

```
> equ2;
```

$$e^t = e^t$$

```
> #and it also checks for the second set of solutions
> with(linalg);
```

```
Warning, the previous binding of the name GramSchmidt has been removed
and it now has an assigned value
```

$[BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol,$
$addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix,$
$charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat,$
$cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge,$
$dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal,$
$exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim,$
$gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert,$
$htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar,$
$iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix,$
$minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog,$
$permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim,$
$rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix,$
$subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose,$
$vandermonde, vecpotent, vectdim, vector, wronskian]$

```
> W:= t->det([[exp(2*t),(2/3)*exp(2*t)],[exp(t),exp(t)]]);
```

$$\mathrm{W} := t \to \det([[e^{(2\,t)}, \frac{2}{3}\,e^{(2\,t)}], [e^t, e^t]])$$

```
> W(0);  #since the Wronskian is nonzero at 0, the functions are
> linearly independent.
```

$$\frac{1}{3}$$

```
> c1:=evaln(c1); c2:=evaln(c2);x:=t->c1*exp(2*t)+c2*exp(t);
```

$$c1 := c1$$

$$c2 := c2$$

$$x := t \to c1\,e^{(2\,t)} + c2\,e^t$$

```
> y:=t->c1*(2/3)*exp(2*t)+c2*exp(t);
```

$$y := t \to \frac{2}{3}\,c1\,e^{(2\,t)} + c2\,e^t$$

```
> solve({x(0)=1,y(0)=2});
```

$$\{c1 = -3,\ c2 = 4\}$$

```
> #that might seem too much like magic.

> x(0)=1;
```

$$c1 + c2 = 1$$

```
> y(0)=2;
```

$$\frac{2}{3}\,c1 + c2 = 2$$

```
> #now solve these linear equations for c1 and c2 to get
> #the answer given above.
```

```
> c1:=-3; c2:=4; matrix(2,1,[[x(t)],[y(t)]]);
```

$$c1 := -3$$

$$c2 := 4$$

$$\begin{bmatrix} -3\,e^{(2\,t)} + 4\,e^{t} \\ -2\,e^{(2\,t)} + 4\,e^{t} \end{bmatrix}$$

```
> #the final answer in vector form.

> equ1;
```

$$-6\,e^{(2\,t)} + 4\,e^{t} = -6\,e^{(2\,t)} + 4\,e^{t}$$

```
> equ2;
```

$$-4\,e^{(2\,t)} + 4\,e^{t} = -4\,e^{(2\,t)} + 4\,e^{t}$$

```
> #and the final answer does satisfy the original equations!

> #Problem 9

> with(LinearAlgebra);
```

```
Warning, the assigned name GramSchmidt now has a global binding
```

[*Add*, *Adjoint*, *BackwardSubstitute*, *BandMatrix*, *Basis*, *BezoutMatrix*, *BidiagonalForm*, *BilinearForm*, *CharacteristicMatrix*, *CharacteristicPolynomial*, *Column*, *ColumnDimension*, *ColumnOperation*, *ColumnSpace*, *CompanionMatrix*, *ConditionNumber*, *ConstantMatrix*, *ConstantVector*, *CreatePermutation*, *CrossProduct*, *DeleteColumn*, *DeleteRow*, *Determinant*, *DiagonalMatrix*, *Dimension*, *Dimensions*, *DotProduct*, *Eigenvalues*, *Eigenvectors*, *Equal*, *ForwardSubstitute*, *FrobeniusForm*, *GaussianElimination*, *GenerateEquations*, *GenerateMatrix*, *GetResultDataType*, *GetResultShape*, *GivensRotationMatrix*, *GramSchmidt*, *HankelMatrix*, *HermiteForm*, *HermitianTranspose*, *HessenbergForm*, *HilbertMatrix*, *HouseholderMatrix*, *IdentityMatrix*, *IntersectionBasis*, *IsDefinite*, *IsOrthogonal*, *IsSimilar*, *IsUnitary*, *JordanBlockMatrix*, *JordanForm*, *LA_Main*, *LUDecomposition*, *LeastSquares*, *LinearSolve*, *Map*, *Map2*, *MatrixAdd*, *MatrixInverse*, *MatrixMatrixMultiply*, *MatrixNorm*, *MatrixScalarMultiply*, *MatrixVectorMultiply*, *MinimalPolynomial*, *Minor*, *Multiply*, *NoUserValue*, *Norm*, *Normalize*, *NullSpace*, *OuterProductMatrix*, *Permanent*, *Pivot*, *QRDecomposition*, *RandomMatrix*, *RandomVector*, *Rank*, *ReducedRowEchelonForm*, *Row*, *RowDimension*, *RowOperation*, *RowSpace*, *ScalarMatrix*, *ScalarMultiply*, *ScalarVector*, *SchurForm*, *SingularValues*, *SmithForm*, *SubMatrix*, *SubVector*, *SumBasis*, *SylvesterMatrix*, *ToeplitzMatrix*, *Trace*, *Transpose*, *TridiagonalForm*, *UnitVector*, *VandermondeMatrix*, *VectorAdd*, *VectorAngle*, *VectorMatrixMultiply*, *VectorNorm*, *VectorScalarMultiply*, *ZeroMatrix*, *ZeroVector*, *Zip*]

```
> x:=evaln(x); y:=evaln(y);
```
$$x := x$$
$$y := y$$
```
> equ1:=D(x)(t)=5*x(t)+4*y(t);
```
$$equ1 := \mathrm{D}(x)(t) = 5\,\mathrm{x}(t) + 4\,\mathrm{y}(t)$$
```
> equ2:=D(y)(t)=-6*x(t)-5*y(t);
```
$$equ2 := \mathrm{D}(y)(t) = -6\,\mathrm{x}(t) - 5\,\mathrm{y}(t)$$
```
> dsolve({equ1,equ2});
```
$$\{\mathrm{x}(t) = \_C1\; e^{(-t)} + \_C2\; e^t,\ \mathrm{y}(t) = -\frac{3}{2}\_C1\; e^{(-t)} - \_C2\; e^t\}$$
```
> #Perhaps that was a little fast...
> lambda:=evaln(lambda);
```
$$\lambda := \lambda$$
```
> A:=Matrix(2,2,[[5,4],[-6,-5]]);
```
$$A := \begin{bmatrix} 5 & 4 \\ -6 & -5 \end{bmatrix}$$
```
> A_lambda:=Matrix(2,2,[[5-lambda,4],[-6,-5-lambda]]);
```
$$A\_lambda := \begin{bmatrix} 5-\lambda & 4 \\ -6 & -5-\lambda \end{bmatrix}$$
```
> det(A_lambda);
```
$$-1 + \lambda^2$$
```
> solve(det(A_lambda)=0);
```
$$1,\ -1$$
```
> lambda:=-1;
```
$$\lambda := -1$$
```
> MatrixMatrixMultiply(A_lambda,Matrix(2,1,[[a],[b]])); #this
vector
> will be zero iff [[a],[b]] is an eigenvector for this value of
> lambda.
```
$$\begin{bmatrix} 6\,a + 4\,b \\ -6\,a - 4\,b \end{bmatrix}$$
```
> solve({a=1,6*a+4*b = 0});
```
$$\{a = 1,\ b = \frac{-3}{2}\}$$
```
> #so the eigenvector for lambda = -1 is [[1],[-3/2]].
> lambda:=1;
```
$$\lambda := 1$$
```
> MatrixMatrixMultiply(A_lambda,Matrix(2,1,[[a],[b]])); #this
vector
> will be zero iff [[a],[b]] is an eigenvector for this value of
> lambda.
```

$$\begin{bmatrix} 4a+4b \\ -6a-6b \end{bmatrix}$$

```
> solve({a=1,4*a+4*b = 0});
```

$$\{b=-1,\ a=1\}$$

```
> #so the eigenvector for lambda = 1 is [[1],[-1]].

> c1*exp(t)*Matrix(2,1,[[1],[-1]])+c2*exp(-t)*Matrix(2,1,[[1],[-3/2]]);
```

$$e^t \begin{bmatrix} -3 \\ 3 \end{bmatrix} + e^{(-t)} \begin{bmatrix} 4 \\ -6 \end{bmatrix}$$

```
> c1:=evaln(c1); c2:=evaln(c2);
```

$$c1 := c1$$
$$c2 := c2$$

```
> x:=t->c1*exp(t)+c2*exp(-t);
```

$$x := t \rightarrow c1\ e^t + c2\ e^{(-t)}$$

```
> y:=t->-c1*exp(t)-(3/2)*c2*exp(-t);
```

$$y := t \rightarrow -c1\ e^t - \frac{3}{2}\ c2\ e^{(-t)}$$

```
> equ1;
```

$$c1\ e^t - c2\ e^{(-t)} = c1\ e^t - c2\ e^{(-t)}$$

```
> equ2;
```

$$-c1\ e^t + \frac{3}{2}\ c2\ e^{(-t)} = -c1\ e^t + \frac{3}{2}\ c2\ e^{(-t)}$$

```
> #so the purported general solution does satisfy the original
> equation.
> x:=evaln(x); y:=evaln(y);c1:=evaln(c1);
> c2:=evaln(c2);lambda:=evaln(lambda);
```

$$x := x$$
$$y := y$$
$$c1 := c1$$
$$c2 := c2$$
$$\lambda := \lambda$$

```
> equ1:=D(x)(t)=x(t)-y(t);
```

$$equ1 := \mathrm{D}(x)(t) = \mathrm{x}(t) - \mathrm{y}(t)$$

```
> equ2:=D(y)(t)=x(t)+y(t);
```

$$equ2 := \mathrm{D}(y)(t) = \mathrm{x}(t) + \mathrm{y}(t)$$

```
> dsolve({equ1,equ2});
```

$$\{\mathrm{y}(t) = -e^t\,(\_C1\cos(t) - \_C2\sin(t)),\ \mathrm{x}(t) = e^t\,(\_C1\sin(t) + \_C2\cos(t))\}$$

```
> #Perhaps that was a little fast...
```

```
> A:=Matrix(2,2,[[1,-1],[1,1]]);
```

$$A := \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

```
> lambda:=evaln(lambda);
```

$$\lambda := \lambda$$

```
> A_lambda:=Matrix(2,2,[[1-lambda,-1],[1,1-lambda]]);
```

$$A\_lambda := \begin{bmatrix} 1-\lambda & -1 \\ 1 & 1-\lambda \end{bmatrix}$$

```
> det(A_lambda);
```

$$2 - 2\,\lambda + \lambda^2$$

```
> solve(det(A_lambda)=0);
```

$$1 + I,\, 1 - I$$

```
> lambda:=1+I;
```

$$\lambda := 1 + I$$

```
> MatrixMatrixMultiply(A_lambda,Matrix(2,1,[[a],[b]])); #this
vector
> will be zero iff [[a],[b]] is an eigenvector for this value of
> lambda.
```

$$\begin{bmatrix} -I\,a - b \\ a - I\,b \end{bmatrix}$$

```
> solve({a=1,-I*a-b = 0});
```

$$\{b = -I,\, a = 1\}$$

```
> #so the eigenvector for lambda = 1+I is [[1],[-I]].
> #the eigenvector for 1-I will be [[1],[I]].
> Sol:=c1*exp((1+I)*t)*Matrix(2,1,[[1],[-I]])+c2*exp((1-I)*t)*Matrix(2,
> 1,[[1],[I]]);
```

$$Sol := c1\, e^{((1+I)\,t)} \begin{bmatrix} 1 \\ -I \end{bmatrix} + c2\, e^{((1-I)\,t)} \begin{bmatrix} 1 \\ I \end{bmatrix}$$

```
> Sol:=simplify(Sol);Sol:=map(evalc,Sol); #this converts the complex
> exponentials to trig form
```

$$Sol := \begin{bmatrix} c2\, e^{((1-I)\,t)} + c1\, e^{((1+I)\,t)} \\ I\, c2\, e^{((1-I)\,t)} - I\, c1\, e^{((1+I)\,t)} \end{bmatrix}$$

$$Sol := \begin{bmatrix} c2\, e^t \cos(t) + c1\, e^t \cos(t) + I\,(-c2\, e^t \sin(t) + c1\, e^t \sin(t)) \\ c2\, e^t \sin(t) + c1\, e^t \sin(t) + I\,(c2\, e^t \cos(t) - c1\, e^t \cos(t)) \end{bmatrix}$$

```
> c1:=1/2; c2:=1/2; Sol1:=Sol;
```

$$c1 := \frac{1}{2}$$

$$c2 := \frac{1}{2}$$

$$Sol1 := \begin{bmatrix} e^t \cos(t) \\ e^t \sin(t) \end{bmatrix}$$

```
> c1:=1/(2*I); c2:=-1/(2*I); Sol2:=Sol;
```

$$c1 := \frac{-1}{2} I$$

$$c2 := \frac{1}{2} I$$

$$Sol2 := \begin{bmatrix} e^t \sin(t) \\ -e^t \cos(t) \end{bmatrix}$$

I found these real solutions by direct computation from the complex solutions; to use the formula

in the book, lambda = 1+I so alpha = 1 and beta =1, and the eigenvector [[1][-I]] breaks up

into [[1],[0]]+[[0],[-1]]I so v1 is (1,0)^T and v2 is (0,-1)^T (switching back from Maple notation

to something more usual).

```
> c1:=evaln(c1); c2:=evaln(c2);
```

$$c1 := c1$$

$$c2 := c2$$

```
> x:=t->c1*exp(t)*cos(t)+c2*exp(t)*sin(t);
```

$$x := t \to c1\ e^t \cos(t) + c2\ e^t \sin(t)$$

```
> y:=t->c1*exp(t)*sin(t)-c2*exp(t)*cos(t);
```

$$y := t \to c1\ e^t \sin(t) - c2\ e^t \cos(t)$$

```
> equ1;evalb(equ1);
```

$$c1\ e^t \cos(t) - c1\ e^t \sin(t) + c2\ e^t \sin(t) + c2\ e^t \cos(t) = \\ c1\ e^t \cos(t) - c1\ e^t \sin(t) + c2\ e^t \sin(t) + c2\ e^t \cos(t)$$

$$true$$

```
> equ2;evalb(equ2);
```

$$c1\ e^t \sin(t) + c1\ e^t \cos(t) - c2\ e^t \cos(t) + c2\ e^t \sin(t) = \\ c1\ e^t \sin(t) + c1\ e^t \cos(t) - c2\ e^t \cos(t) + c2\ e^t \sin(t)$$

$$true$$

```
> #so the purported general solution does satisfy the original
> equation.
```