Math 333 Practice Test 4

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April 28, 2003

Use of a calculator is permitted on this exam (and will be permitted on the real exam), but be sure to provide work when it is requested. In general, if you use your calculator show the setup for your calculations on your paper.

This is longer than the actual exam. Some problems you are asked to solve in full here may be given partially solved in the exam in its final form. Some kinds of problems may not appear, and of course problems may appear which have no exact model here.

The exam will be given on Wednesday and Friday this week, in a similar format to the last exam.

A solution set for this practice exam will be made available on Tuesday.

1. For each computation with matrices and/or vectors, either compute the answer (showing hand calculations – calculator answers not good for full credit) or state that it is undefined and why.

   (a)
   $$\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

   (b)
   $$\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ 5 \end{pmatrix}$$

   (c)
   $$\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & -2 & 0 \\ -2 & 2 & 1 \end{pmatrix}$$

(d)

$$\begin{pmatrix} 1 & -2 & 0 \\ -2 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$$

2. Solve the systems of equations using row operations and back-solving. Describe the row operations you use and show the matrices on your paper; you may use a calculator to carry them out.

   Give a completely general solution in vector notation.

   (a)

$$x + y + z = 4$$
$$x - 2y + z = 1$$
$$2x + 3y + 4z = 13$$

   (b)

$$x - y + z = 4$$
$$2x - y - z = 1$$

3. Compute the inverse of the matrix

$$\begin{pmatrix} 1 & 3 \\ 5 & 1 \end{pmatrix}$$

   using row operations and showing all steps. You may use a calculator to carry out the row operations, but you must describe the row operations and show the matrices on your paper.

   Use a matrix calculation with this inverse matrix to solve the system of equations

$$x + 3y = 6$$
$$5x + y = 16$$

   You may use a calculator to carry out this computation, but you must set it up on your paper.

4. The set of vectors

$$\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 8 \\ -8 \\ -6 \end{pmatrix}$$

is linearly dependent. Find a nontrivial linear combination of these vectors which is equal to the zero vector, by appropriate matrix methods.

5. Calculate the determinant of the matrix using any method you prefer (but it must be a hand calculation, which you must show on your paper; you may use a calculator only to check). Decide using the determinant whether the matrix has a nontrivial nullspace, and if it does, find a basis for the nullspace and state the dimension of the nullspace.

$$\begin{pmatrix} 1 & -2 & 3 \\ -1 & -1 & 2 \\ 3 & 0 & -1 \end{pmatrix}$$

6. Write the third-order equation

$$y''' - ty'' + 2y' - \sin(t)y = \cos(t)$$

as a system of first-order differential equations.

7. A system of tanks with various pipes transferring water with different percentages of HCl at different rates is shown. Set up a system of differential equations and initial value problem whose solution will describe the amount of salt in each tank at time t.

8. Verify that

$$\begin{pmatrix} e^{2t} \\ \frac{2}{3}e^{2t} \end{pmatrix}$$

and

$$\begin{pmatrix} e^{t} \\ e^{t} \end{pmatrix}$$

are solutions of

$$x' = 4x - 3y$$
$$y' = 2x - y$$

Verify that these are linearly independent solutions using the Wronskian.

Find the solution to this equation satisfying the initial condition

$$x(0) = 1; y(0) = 2$$

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9. Solve each of the systems of linear differential equations with constant coefficients using the method of eigenvectors and eigenvalues.

You must show hand calculations to find the eigenvectors and eigenvalues for full credit.

If the eigenvalues are complex, you need to give real solutions to the equations.

(a)

$$x' = 5x + 4y$$
$$y' = -6x - 5y$$

(b)

$$x' = x - y$$
$$y' = x + y$$