7.3 Problem Set

Dr. Holmes

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Solve the following problems using trigonometric substitutions (showing all details of the substitutions, and following any additional instructions).

1. You can find the antiderivative
   \[ \int x\sqrt{4-x^2}\,dx \]
   by using an easy substitution (do it). Now solve the problem using a trigonometric substitution and verify that your answer is equivalent.

2. Set up the familiar problem of finding the area of a circle of radius \( r \) as an integration problem and solve it using a trigonometric substitution (you may assume that it is centered at the origin, and you may exploit symmetries to simplify the problem).

3. Evaluate the integrals using trigonometric substitution:
   (a) \[ \int \frac{x^2}{\sqrt{1-x^2}}\,dx \]
   (b) \[ \int \frac{1}{x\sqrt{x^2-1}}\,dx \]
      (You might recognize this one, but go ahead and carry out the substitution).
   (c) \[ \int \frac{x^2}{\sqrt{9+x^2}}\,dx \]