11.1 Problem Set

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I completely redid problem 1 after seeing what happened in the class discussion. I also changed problem 2 part c, which happened to be the same as an example in the section, to something simpler (and probably easier).

1. Write the next four terms and give a description for the \( n \)th term of each of the following sequences. Each of them has the indexing starting with 1. The first two are described by formulas and the last one has a recursive rule.

(a) 
\[ 3, 7, 11, 15, \ldots \]

(b) 
\[ \frac{-4 \cdot 7}{5 \cdot 25}, \frac{-10}{125}, \frac{13}{625}, \ldots \]

(c) 
\[ 2, 3, 7, 13, 27, 53, \ldots \]

2. Determine the limit of each of the following sequences or explain why it diverges.

(a) 
\[ a_n = \frac{1 - n^2}{2n^2 + 1} \]

(b) 
\[ b_n = \arctan \left( \frac{n^3}{n^2 + 1} \right) \]
(c) 

\[ c_n = \frac{(n+1)!}{n!} \]

(write out a few terms of this sequence and you should see what is happening).

(d) 

\[ \cos(n) \]

(look at what this does on a calculator (use radian mode))

3. What is the limit of the sequence whose first term is 1 and which satisfies

\[ x_{n+1} = \frac{1}{2}(x_n + \frac{3}{x_n}) \]?

You may assume in your argument that there is actually a limit. Your limit must be stated in exact form with a supporting argument. Hint: look at problem 52 in the book: use the fact stated in part a.

4. Prove that

\[ \lim_{n \to \infty} \frac{n + 2}{n} = 1 \]

using the official definition of limit.