

Math 187 Test II

Dr. Holmes

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This exam will begin at 11:40 am and end at 12:35 pm. If your paper is requested at 12:35 pm, please surrender it promptly and leave the room so as not to obstruct the arrival of the next class.

A plain scientific calculator with no graphing or symbolic computation capability is allowed (and might be needed on this or a subsequent exam); other calculators are not allowed, and one may not use a PDA or cell phone as a calculator under any circumstances. Books, notes, and neighbors are to remain firmly closed.

The weight of the questions may be modified to reflect class performance on individual questions; you may assume that the weights are approximately equal. It is not worthwhile to pursue extra credit opportunities unless you are finished with the rest of the exam.

1. One-to-one, onto, etc.

- (a) State what it means for a function f from A to B to be one-to-one. State what it means for a function f from A to B to be onto the target B .

- (b) How can I modify the domain and/or target of the function $f(x) = x^2$ with domain \mathcal{R} (the set of reals) and target \mathcal{R} so as to make it one-to-one?

How can I modify the domain and/or target of the same function to make it onto? (Your corrected domain and/or target should still be large sets.)

Draw appropriate graphs to illustrate why the changes you suggest work.

2. Compositions and inverses. Do one of the two parts. If you do both, your best work will count.

- (a) The functions f and g are given as sets of ordered pairs. One of $f \circ g \circ f^{-1}$ and $g \circ f \circ g^{-1}$ is defined and the other is not. Explain why the one which is undefined is undefined, and calculate the one which is defined and present it as a set of ordered pairs. Illustrate your work with appropriate arrow diagrams.

$$f = \{(1, 2), (2, 3), (3, 1), (4, 4)\}$$

$$g = \{(1, 2), (2, 2), (3, 4), (4, 1)\}$$

- (b) The function $h(x) = \frac{1-x}{2+x}$ from $\mathcal{R} - \{-2\}$ to \mathcal{R} is one-to-one. Determine the inverse of h and explain how we would have to modify the domain or target of h to make h onto as well.

3. one-to-one correspondence and cardinality. Note that part b is extra credit.

(a) Present a bijection from $\{1, 2, 3\}$ to $\{a, b, c\}$ (where a, b, c are distinct objects).

(b) (**extra credit**) How many bijections are there from $\{1, 2, 3\}$ to $\{a, b, c\}$? List them.

(c) Present a bijection from the set of integers \mathcal{Z} to the set of pairs of natural numbers $\mathcal{N} \times \mathcal{N}$. You need only present a pattern, not a formula, but you must present sufficient evidence that you know what the pattern is (a diagram would help) and you must actually present a partial list of values of the bijection (*just* the diagram is not enough).

4. Division algorithm: for the following values of a and b , present numbers q and r (quotient and remainder) such that $a = bq + r$ and $0 \leq r < |b|$.

(a) $a = 10; b = 3$

(b) $a = -10; b = 3$

(c) $a = 3; b = 10$

(d) $a = -3; b = 10$

5. Bases

- (a) Convert the base ten numeral 213 to base 8. Show work (if your calculator does this conversion, you may use it only to check; you must show the full calculation we have used in class). Then convert it to base 2 without any further work in base ten.
- (b) Convert the base ten numerals 13 and 19 to base six.
Add them in base six. Check that you have the correct value for the sum by converting back to decimal.
Multiply them in base six. Check that you have the correct value for the product by converting back to decimal.

6. Find the greatest common divisor of 177 and 72 using the Euclidean algorithm. Use your calculation of the gcd to find integers m and n such that $177m + 72n = \gcd(177, 72)$. Use the same table format used in the book and in my board examples.

7. Draw the Hasse diagram for the partial order of divisibility on the proper divisors of 30. Does this partial order have a minimum? Does it have minimal elements? Does it have a maximum? Does it have maximal elements?

8. Proofs and derivations. Do one of the following. If you do more than one, your best work will count. If you do more than one well, some extra credit may be possible. These are all proofs I did in class.
- (a) Prove that if f is a one-to-one function from A to B and g is a one-to-one function from B to C , then $g \circ f$ is a one-to-one function from A to C .
 - (b) Prove that the interval $(0, 1)$ in the reals is uncountable. Just drawing the illustration is not enough; you need to write some English.
 - (c) Prove that if a and b are relatively prime natural numbers and $a|bx$, then $a|x$ must follow. Hint: use the fact that there must be integers m and n such that $ma + nb = 1$ to rewrite x in a way which makes it clear that it must be divisible by a . Provide all needed supporting comments.