PREPARATION FOR THE FINAL EXAM MATH 170, FALL 2006

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Everything that has been done so far could appear on the final exam. It would not harm if you had a look at the old exams. Some subjects are especially suitable for exam problems: computing limits, taking derivatives, related rates, finding local extremes, finding antiderivatives, computing definite integrals.

Important concepts are continuity, derivative and differentiability, definite integral, indefinite integral.

The main theorems are Intermediate Value Theorem, Mean Value Theorem, the connection between local extremes and zeros of the derivative, Fundamental Theorem of Calculus.

The following problems focus a bit more on proofs than the actual exam will. On the final there will be more or less straight forward computations, proofs, word problems and questions asking for definitions.

Problem 1. Show that there is a positive real number a such that $a^2 = 3$. In other words, show that $\sqrt{3}$ exists.

Problem 2. Show that

$$\lim_{x \to 0^+} \frac{1}{x^2} = \infty.$$

Problem 3. Show, using the definition of the derivative as a limit, that for every $x \in \mathbb{R}$ we have $(2x^3 + 1)' = 6x^2$.

Problem 4. Show that for every real number c the equation $x^4 + 4x + c = 0$ has at most two real roots.

Problem 5. Show that two antiderivatives of the same function only differ by a constant.

Problem 6. Find the limit.

a) $\lim_{x \to \infty} \frac{x^4 - 7x + 1}{3x^4 + 10000x}$ b) $\lim_{x \to 0} (\sin x \cdot \ln x)$

Problem 7. Find the derivative of the given function.

a) $f(x) = \cosh(\arcsin x)$ b) $f(x) = x^{\ln x}$

Problem 8. Find an antiderivative of the given function.

a) $f(x) = e^{3x} + x^2$ b) $f(x) = \frac{3}{x} + \cos x$

Problem 9. Let f be a continuous function defined of the interval $[a, b]$. Write down the formal definition of the real number $\int_a^b f(x)dx$.

Problem 10. State the Fundamental Theorem of Calculus in one of its two forms. The book calls the two forms Part 1 and Part 2, but really they are the same theorem.

Problem 11. The speed of a car at time $t \in [0, 1]$ is given by $f(t) = -90t^2 + 90t$. The time is given in hours, the speed in miles per hour. What is the maximal speed? What is the distance traveled at time $t = 1$?