1. Which of the following guarantee that the matrix $A \in \mathbb{R}^{n \times n}$ has an inverse? Circle all that apply.

(a) $A$ has a complete set of eigenvectors
(b) The determinant of $A$ is not zero,
(c) $A$ has at least one zero pivot,
(d) There is exactly one solution to $Ax = b$.
(e) The dimension of the nullspace of $A$ is 0,
(f) The columns of $A$ span all of $\mathbb{R}^n$.
(g) $\text{rank}(A) = n$.
(h) $A$ has at least one non-zero eigenvalue,
(i) $A$ is diagonalizable,
(j) The reduced row echelon form of $A$ is $R = I$.
(k) $A$ has $n$ (non-zero) singular values.
(l) $A$ is similar to a diagonal matrix.
(m) $A$ has all positive eigenvalues.
(n) $A$ has real eigenvalues.
(o) $A$ is symmetric.
(p) $x^T A x > 0$ for all non-zero $x \in \mathbb{R}^n$.

2. How many eigenvectors do the following matrices have? Don’t do any computations, but explain your reasoning.

(a) \[
\begin{bmatrix}
-2 & 1 & 0 \\
0 & -2 & 1 \\
0 & 0 & -2
\end{bmatrix}
\quad (b) \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad (c) \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}
\]

(d) \[
\begin{bmatrix} 3 & 0 \\ 0 & -7 \end{bmatrix}, \quad (e) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (f) \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

(a) Is a non-trivial Jordan block with 2 missing eigenvectors. Only has one eigenvector.
(b) 2 evces - symmetric
(c) Symmetric \Rightarrow 2 evces.
(d) 2 evces
(e) 3 evces - symmetric
(f) 2 missing evces \Rightarrow 4 total \Rightarrow only 2 evces.