These homework problems are to be turned in and will be graded for credit. Turn in your work on separate pages, using this as a cover sheet. Please staple your work together. For full credit, you must show all of your work.

1. Find a basis for each of these subspaces in $\mathbb{R}^4$.
   (a) All vectors whose components are equal.
   (b) All vectors whose components add to zero.
   (c) All vectors that are perpendicular to $(1, 1, 0, 0)$ and $(1, 0, 1, 1)$.
   (d) The column space and the nullspace of the $4 \times 4$ identity matrix $I$.

2. Given a linearly independent set of vectors $u_1, u_2, u_3$, all in $\mathbb{R}^3$, construct a new set of vectors $v_1, v_2, v_3$ as follows:
   
   \[ v_1 = -2u_1 + u_2 \]
   \[ v_2 = u_1 - 2u_2 + u_3 \]
   \[ v_3 = u_2 - 2u_3 \]

   Show that $v_1, v_2, v_3$ are linearly independent.

3. If $V$ is the subspace spanned by $(0, 1, 1)$ and $(2, 0, 1)$, find a matrix $A$ that has $V$ as its row space. Find a matrix $B$ that has $V$ as its nullspace.

4. If the entries of a $4 \times 4$ matrix are chosen randomly between 0 and 1, what are the most likely dimensions of the four subspaces? What if the matrix is $4 \times 7$?

5. Add the extra column $b$ and reduce $A$ to echelon form.

   \[
   \begin{bmatrix}
   A & b
   \end{bmatrix} = \begin{bmatrix}
   1 & 2 & b_1 \\
   3 & 4 & b_2 \\
   4 & 6 & b_3
   \end{bmatrix}
   \]

   From the $b$ column after elimination, read of $m - r$ basis vectors in the left nullspace. Show that those $y$’s are the combinations of rows that give zero rows.

6. Construct $A = uv^T + wz^T$ (where $u$ and $v$ are rank 1 matrices) are who column space has basis $(1, 2, 4), (2, 2, 1)$ and whose row space has basis $(1, 0), (1, 1)$. Write $A$ as a $3 \times 2$ matrix times a $2 \times 2$ matrix.