Homework #5
Math 301, Spring 2013
Due Wednesday, March 6, 2013

These homework problems are to be turned in and will be graded for credit. Turn in your work on separate pages, using this as a cover sheet. Please staple your work together. For full credit, you must show all of your work.

1. What permutation makes $PA$ upper triangular? Which permutation matrices $P_1$ and $P_2$ make $P_1AP_2$ lower triangular?

\[
A = \begin{bmatrix}
0 & 0 & 6 \\
1 & 2 & 3 \\
0 & 4 & 5 \\
\end{bmatrix}
\]

2. If $A = A^T$ and $B = B^T$, which of these matrices are guaranteed to be symmetric?
   (a) $A^T - B^T$
   (b) $(A + B)(A - B)$
   (c) $ABA$
   (d) $ABAB$

3. The matrix $A = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$ is a “vector” in vector space $\mathbf{M}$ of all $2 \times 2$ matrices. Write down the zero vector in this space, the vector $\frac{1}{2}A$, and the vector $-A$. What matrices are in the smallest subspace containing $A$?

4. Describe the column space (lines or spaces) of these matrices

\[
A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 0 \end{bmatrix}
\]

5. Construct a 3 by 3 matrix whose column space contains $(1, 1, 0)$ and $(1, 0, 1)$ but not $(1, 1, 1)$. Construct a 3 by 3 matrix whose column space is only a line.