1. Given two vectors $u = (-2, 2)$ and $v = (3, 1)$, do the following.

(a) Sketch the two vectors. Make your sketch large enough so you can add a few more vectors.

(b) Add to your sketch a unit vector $e_v$ parallel to $v$.

(c) Add to your sketch the projection $u \parallel$ of $u$ along $v$. Write this projected vector both in terms of $v$ and in terms of $e_v$. Use both forms to compute the length of the projected vector.

(d) The dot product is *commutative*. In other words, $u \cdot v = v \cdot u$. But does this mean that the projection of $v$ along $u$ is the same as the projection of $u$ along $v$? Write the projection of $v$ along $u$ in terms of both $u$ and $e_u$ (a unit vector parallel to $u$). What is the length of the projected vector $v \parallel$?

(e) Argue geometrically that the length of the projection of $e_v$ along $e_u$ is the same as the length of the projection of $e_u$ along $e_v$. Are these the same vectors? Draw a second sketch illustrating these two unit vectors and their projections onto each other.
2. Let $\mathbf{u} = (1, 2, -4)$ and $\mathbf{v} = (-3, -2, 1)$.

(a) Find a vector $\mathbf{w}$ that is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.
(b) Verify that $\mathbf{w} \cdot \mathbf{u} = 0$ and $\mathbf{w} \cdot \mathbf{v} = 0$.
(c) What is the length of $\mathbf{w}$? Can you relate the length of $\mathbf{w}$ to the dot product $\mathbf{u} \cdot \mathbf{v}$? Write down a general formula relating the magnitude $\|\mathbf{u} \times \mathbf{v}\|$ to $\|\mathbf{u}\|$, $\|\mathbf{v}\|$, and $\mathbf{u} \cdot \mathbf{v}$.
(d) Write down an equation for all points $\mathbf{P} = (x, y, z)$ such that $\mathbf{w} \cdot \overrightarrow{PQ} = 0$, for $Q = (-1, 3, 4)$.
(e) Describe in words the set of points satisfying the equation you found above.