Math 275 : In-class exercise #11

Set up and evaluate the following surface integrals.

1. Suppose the vector field $F$ is given as $F = \nabla V$, where $V(x, y) = 2z^2 \sin(x + y)$. Show that $\nabla \times F = 0$.

2. Verify Green’s Theorem for the vector field $F = (e^{2x+y}, e^{-y})$, over the triangle with vertices (0, 0), (1, 0) and (1, 1).
3. Verify Stokes Theorem for the vector field $F = (2xy, x, y + z)$ over the surface described by $z = 1 - x^2 - y^2$, for $x^2 + y^2 \leq 1$. What is the circulation around the boundary of the surfaces described by $z = \sqrt{1 - x^2 - y^2}$ or $z = 0$, also for $x^2 + y^2 \leq 1$?

4. Verify the Divergence Theorem for the vector field $F = (x, 0)$ over the region defined by the intersection of the unit square (with lower left corner (0,0) and upper right corner (1,1)) and the region below the line $5x - 12y + 4 = 0$. How does the Divergence Theorem help you compute the area of the region?

5. Use the Divergence Theorem to compute the volume of the region defined as the intersection of the unit cube (with lower left corner at (0,0,0)) and the region below the plane $8z - 2x = 3$. 