1. Write down a set of parametric equations for the line segment between the points \((2, -1, 2)\) and the point \((4, 3, -6)\).

2. A snail travels along the trajectory parameterized as \(c(t) = (3 - t, 4 + 2t, -1 + 3t)\), where the snail’s position \((x(t), y(t), z(t))\) is given in centimeters and the \(t\) is the time, in seconds. How long does it take the snail to travel 10 centimeters? You can assume that the snail starts at time \(t = 0\).

3. Two vectors \(\mathbf{u}\) and \(\mathbf{v}\) have the following properties:
   \[
   \|
   \mathbf{u}\|
   = 5
   \|
   \mathbf{v}\|
   = 7
   \mathbf{u} \cdot \mathbf{v} = -2
   \]
   What is \(\|
   \mathbf{u} \times \mathbf{v}\|\)?

4. Find \(\|2\mathbf{e} - 3\mathbf{f}\|\) assuming that \(\mathbf{e}\) and \(\mathbf{f}\) are unit vectors and that \(\|\mathbf{e} + \mathbf{f}\| = \sqrt{3/2}\).

5. Find the equation of the plane passing through the three points \((5, 1, 1)\), \((1, 1, 2)\), and \((2, 1, 1)\).

6. Find the equation of the plane that contains the line \(r(t) = (1 - t, 3 + 5t, -4t)\) and the point \((1, 1, 2)\).

7. Find parametric equations for the intersection of the planes \(2x + y - 3z = 0\) and \(x + y = 1\).

8. Use the sine and the cosine to parameterize the intersection of the surfaces \(x^2 + y^2 = 1\) and \(z = 4x^2\).
9. Determine whether \( \mathbf{r}_1(t) \) and \( \mathbf{r}_2(t) \) collide or intersect:

\[
\mathbf{r}_1(t) = (t, t^2, t^3), \quad \mathbf{r}_2(t) = (4t + 6, 4t^2, 7 - t)
\]

10. Find a parameterization of the tangent line to the curve \( \mathbf{r}(t) = (1 - t^2, 5t, 2t^3) \) at the point \( t = 2 \).

11. Find an arc length parameterization of the circle in the plane \( z = 9 \) with the radius 4 and center \( (1, 4, 9) \).

12. Find an arc length parameterization of the line \( y = 4x + 9 \).

13. Show that the curvature of a circle of radius \( R \) is given by \( \kappa = 1/R \).

14. Show that the curvature of a line is 0.

15. Find a formula for the curvature at a point on a graph \( y = f(x) \) in the plane.

16. Find the decomposition of \( \mathbf{a}(t) \) into tangential and normal components for the curve \( \mathbf{r}(t) = (e^t, 1 - t) \) at the point \( t = 0 \).

17. Describe the level curves of the function \( f(x, y) = \frac{1}{x^2 + y^2 + 1} \).

18. Find an equation of the tangent plane to the function \( f(x, y) = \frac{x}{\sqrt{y}} \) at the point \( (4, 4) \).
19. Find the points on the graph of \( z = 3x^2 - 4y^2 \) at which the vector \( \mathbf{a} = (3, 2, 2) \) is normal to the tangent plane.

20. Write down a linear approximation to the function \( f(x, y) = x(1 + y)^{-1} \) and use it to estimate the value of \( \frac{7.98}{2.02} \).

21. Use linear approximation to estimate the value of \( \frac{0.98^2}{2.01^3 + 1} \). Compare your answer with the value given by a calculator.

22. Compute the directional derivative of \( f(x, y) = e^{xy} - y^2 \) in the direction of \( \mathbf{v} = (12, -5) \) at the point \( (2, 2) \).

23. Let \( \mathbf{r} = (x, y, z) \) and \( \mathbf{e}_r = \mathbf{r}/||\mathbf{r}|| \). Show that if a function \( f(x, y, z) = F(r) \) depends only on the distance from the origin \( r = \sqrt{x^2 + y^2 + z^2} \), then \( \nabla f = F'(r)\mathbf{e}_r \).

24. Find the critical points of \( f(x, y) = x^3 + y^4 - 6x - 2y^2 \) and use the Second Derivative Test to determine whether they are local minima, local maxima or saddle points (or state that the test fails).

25. Find the critical points of \( f(x, y) = x^2 + y^2 - xy + x \) and use the Second Derivative Test to determine whether they are local minima, local maxima or saddle points (or state that the test fails).

26. Integrate the function \( f(x, y) = \cos(2x + y) \) over the triangle with vertices \((0, 0), (2, 2)\) and \((2, -2)\).

27. Evaluate the integral of \( f(x, y) = x - y \) over the region \( x^2 + y^2 \leq 1, x + y \geq 1 \).
28. Calculate the double integral \( \int \int_D x \sqrt{x^2 + y^2} \, dA \), where \( D \) is the shaded region enclosed by the lemniscate curve \( r^2 = \sin(2\theta) \) (See page 905, problem 24, in your book, for a picture of the figure).

29. Find an appropriate change of variables to evaluate \( \int \int_D (x + y)^2 e^{x^2 - y^2} \, dxdy \) where \( D \) is the square with vertices \((1, 0), (0, 1), (-1, 0), \) and \((0, -1)\).

30. Prove that \( \mathbf{F} = (yz, xz, y) \) is not conservative. Hint: Show that \( \nabla \times \mathbf{F} \neq 0 \).

31. What does the line integral \( \int_a^b \|c'(\tau)\| \, d\tau \) over a curve \( c(t) \), represent? Draw a sketch to illustrate what you mean.

32. What does the surface integral \( \int \int_D \|G_u \times G_v\| \, dudv \), for a surface parameterization \( G(u, v) \) represent? How is this integral analogous to the arc-length integral from Problem 31?

33. Suppose you want to compute the area of the planar region \( D \). What kind of integral would you set up?

34. Compute the line integral \( \int_C f \, ds \) for the function \( f(x, y, z) = 2x^2 + 8z \), over the curve \( c(t) = (e^t, t^2, t) \), \( 0 \leq t \leq 1 \).

35. Compute the line integral \( \int_C \mathbf{F} \cdot ds \) for \( \mathbf{F} = (x^2, xy) \) over part of the circle \( x^2 + y^2 = 9 \) with \( x \leq 0 \), \( y \geq 0 \).
36. The potential $V(x, y) = x \cos(y)$. Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{s}$ where $\mathbf{F} = \nabla V$ and the curve is the upper half of the unit circle, centered at the origin.

37. Calculate $\int \int_S f(x, y, z) \, dS$ for the function $f(x, y, z) = z$, over the surface $y = 9 - z^2$, $0 \leq x \leq 3$, $0 \leq z \leq 3$.

38. Calculate $\int \int_S f(x, y, z) \, dS$ for the surface $G(u, v) = (u, v^3, u + v)$, $0 \leq u \leq 1$ and $0 \leq v \leq 1$ $f(x, y, z) = y$.

39. Calculate $\int \int_S \mathbf{F} \cdot d\mathbf{S}$ for $\mathbf{F} = (xz, yz, z^{-1})$, over the disk of radius 3 at height 4 parallel to the xy-plane, upward-pointing normal.

40. Calculate the integral of $f(x, y, z) = e^z$ over the portion of the plane $x + 2y + 2z = 3$, where $x, y, z \geq 0$.

41. Let $S$ be the surface parametrized by $G(u, v) = \left(2u \sin\left(\frac{v}{2}\right), 2u \cos\left(\frac{v}{2}\right), 3v\right)$ for $0 \leq u \leq 1$ and $0 \leq v \leq \pi$.

   (a) Calculate the tangent vectors $\mathbf{G}_u$ and $\mathbf{G}_v$ and the unit normal vector $\mathbf{N}$ at $P = G(1, \frac{\pi}{3})$.

   (b) Find the equation of the tangent plane at $P$.

   (c) Compute the surface area of $S$.

42. Verify Green’s Theorem for the vector field $\mathbf{F} = (xy, y)$ over the curve $C$ is the unit circle.

43. What is the circulation around any closed loop of conservative vector field?

44. Calculate the circulation of $\mathbf{F} = (xy, x^2 + x)$ around the curve described by the triangle with points $(1, 0)$, $(0, 1)$ and $(-1, 0)$
45. Let \( \mathbf{F} = (0, -z, 1) \). Let \( \mathcal{S} \) be the spherical cap \( x^2 + y^2 + z^2 \leq 1 \), where \( z \geq \frac{1}{2} \). Evaluate \( \int \int_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S} \) directly as a surface integral. Then verify that \( \mathbf{F} = \nabla \times \mathbf{A} \), where \( \mathbf{A} = (0, x, xz) \) and evaluate the surface integral again using Stokes’ Theorem.

46. What is the circulation around a closed surface?

47. What is the correct spelling: “Stoke’s Theorem” or “Stokes’ Theorem”?

48. Let \( \mathcal{C} \) be a closed curve in the plane \( x + y + z = 4 \). The area enclosed by the curve is 16. Let \( \mathbf{F} = (-z^2, 2zx, 4y - x^2) \). Calculate the circulation around \( \mathcal{C} \).

49. Show that the circulation \( \mathbf{F} = (x^2, y^2, z(x^2 + y^2)) \) around any closed curve \( \mathcal{C} \) on the surface of the cone \( z^2 = x^2 + y^2 \) is equal to zero.

50. Compute the divergence of the vector field \( \mathbf{F} = (xy, yz, y^2 - x^3) \).

51. How does the Divergence Theorem imply that the flux of \( \mathbf{F} = (x^2, y - e^z, y - 2zx) \) through a closed surface is equal to the enclosed volume?