Homework #3
Math 567, Fall 2016

Due Friday Dec. 9th

For all problems, turn in your code, and the results of your plots. You may work with a partner on this assignment.

1. Verify that Equation 6.13 (from the red book) results from integrating the piecewise linear solution $\tilde{q}^n(x, t_{n+1})$ from Section 6.4. Then, verify that using the centered slopes in 6.16 lead to the Lax-Wendroff method.

2. Using the code `advect_fv.m` on the course website, fill in details to solve the scalar advection equation using both the upwind scheme, and the Lax-Wendroff scheme.

   $$u_t + au_x = 0$$

subject to periodic boundary conditions.

   - Do not make any assumptions on the sign of the velocity, but take it into account in your scheme, if necessary.
   - The scheme is a “cell-centered” scheme, which for this problem, you can think of as simply a finite-difference scheme defined at the center of each mesh cell, rather than at the nodes (edges).
   - The Lax-Wendroff scheme can be written as the upwind scheme plus a correction term.

Once you have your code working, please turn in a plot of both the first order and second order scheme, at time $T = 5$, and for CFL number $\alpha = 0.1$ and $\alpha = 0.9$.

3. Write a version of problem 2 using the wave-propagation approach. You can start with the code `advect_waveprop.m`. Your main job is to write the Riemann solver for scalar advection. Then, formulate the upwind method (using fluctuations only) and correction terms (using the conservative update). Your results should be essentially the same as what you got for problem 2. The main difference is that using the wave-propagation formulation, we can include “wave limiters”, which act to suppress oscillations resulting from the dispersive nature of the truncation error in the Lax-Wendroff method.

   Produce several plots illustrating the effect of using wave limiters.

4. Solve a system of hyperbolic PDEs (acoustics, 2-way wave equation, or one of the examples of a nonlinear equation). Start with the code `waveprop_systems.m`. Your main job for this problem is to write the Riemann solver, and include the upwind and high-resolution update. Produce one or more plots of your solutions.