Polynomial interpolation

Barycentric Formula for polynomial interpolation
The Lagrange form of the interpolating polynomial is given by

\[ P_n(x) = \sum_{j=0}^{n} \ell_j(x)y_j \]

where

\[ \ell_j(x) = \frac{\prod_{k=0, k \neq j}^{n} (x - x_k)}{\prod_{k=0, k \neq j}^{n} (x_j - x_k)} \]
Lagrange interpolation

However, it is still expensive to compute Lagrange interpolating polynomial.

- Each evaluation of $P_n(x)$ requires $O(n^2)$ flops.

- Adding a new point $(x_{n+2}, y_{n+2})$ to be interpolated requires a whole new computation from scratch.

- The method is numerically unstable.

Still a better method: Barycentric interpolation formula
Barycentric interpolation formula

Define \( \ell(x) = \prod_{k=0}^{n} (x - x_k) \)

Define \( \omega_j = \frac{1}{\prod_{k=0, k \neq j}^{n} (x_j - x_k)} \)

Then \( \ell_j(x) = \ell(x) \frac{\omega_j}{x - x_j} \)

and \( P_n(x) = \ell(x) \sum_{j=0}^{n} \frac{\omega_j}{x - x_j} y_j \)
We have that
\[ \sum_{j=0}^{n} \ell_j(x) = 1 \]

We then have that
\[ \ell(x) = \frac{1}{\sum_{j=0}^{n} \omega_j x - x_j} \]
which leads to the form
\[ P_n(x) = \frac{\sum_{j=0}^{n} \omega_j x - x_j y_j}{\sum_{j=0}^{n} \omega_j x - x_j} \]