Four Fundamental Subspaces

The fundamental Theorem of Linear Algebra

Math 301
Four Fundamental Subspaces

- $C(A)$ is the column space
- $N(A)$ is the nullspace
- $C(A^T)$ is the row space of $A$
- $N(A^T)$ is the left null space of $A$

The left nullspace is the space of all vectors $y$ that satisfy $A^T y = 0$. 

New subspace!
Fundamental Theorem of Linear Algebra

Let $A \in \mathbb{R}^{m \times n}$ with rank $r$.

- The dimension of $C(A)$ is $r$
- The dimension of $N(A)$ is $n - r$
- The dimension of $C(A^T)$ is $r$
- The dimension of $N(A^T)$ is $m - r$

**Remember**

- $C(A) \subset \mathbb{R}^m$
- $N(A) \subset \mathbb{R}^n$
- $C(A^T) \subset \mathbb{R}^n$
- $N(A^T) \subset \mathbb{R}^m$
Four fundamental subspaces

**Question:**

Suppose $A \in \mathbb{R}^{7 \times 14}$ has rank 5. What are the dimensions of the four fundamental subspaces?

**Question:**

The four fundamental subspaces for this matrix are in what four spaces?