

MATH 568 – Homework #6 – optional

due 14 December 2006

- 1) Consider the SOR method for solving the matrix equation

$$A\mathbf{x} = \mathbf{b},$$

where

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

Find the optimal value of the parameter ω associated with the SOR scheme.

- 2) Consider the conjugate-gradient method derived in class. In these exercises, I ask you to derive further properties associated with the method, culminating in more efficient methods of computation for the parameters λ_m and α_m . Prove the following formulae:

a) $\mathbf{d}_m^T \mathbf{r}_{m+1} = 0$

b) $\mathbf{r}_m^T \mathbf{r}_\ell = 0$ for $\ell = 0, 1, \dots, m-1$

c) $\lambda_m = \frac{\mathbf{r}_m^T \mathbf{r}_m}{\mathbf{d}_m^T A \mathbf{d}_m}$

d) $\alpha_m = \frac{\mathbf{r}_{m+1}^T \mathbf{r}_{m+1}}{\mathbf{r}_m^T \mathbf{r}_m}$