

MATH 566 – Homework #9

due 8 November 2007

1) Prove that the equation

$$J(\mathbf{y}) = \frac{1}{2}\|\mathbf{e}\|_A^2 - \frac{1}{2}\|\mathbf{x}\|_A^2,$$

that appears in the middle of page 582, is correct.

2) Do Exercises 7.7.9 and 7.7.12.

3) In Theorem 7.7.14, we worked with the vector space

$$\mathcal{P} = \text{span}\{\mathbf{p}^{(0)}, \dots, \mathbf{p}^{(j)}\}.$$

Prove that the dimension of \mathcal{P} is $j + 1$.

4) Prove Proposition 7.7.22.

5) Let A be the matrix described by Equation (7.1.10) in the text (same as in homework #8). Write a computer program in Matlab to solve

$$A\mathbf{x} = \mathbf{b},$$

via the conjugate gradient method in both its unpreconditioned and preconditioned versions. As before, your preconditioner is the matrix

$$M = \begin{bmatrix} T & & & \\ & T & & \\ & & \ddots & \\ & & & T \end{bmatrix}.$$

Your program should read the data from the same data file as you used in Homework #8, namely `hw8.dat`.

When you get both versions working properly, you should then compare all four methods we have studied to solve this problem, namely steepest decent and conjugate gradient with both absence and presence of preconditioning for each. Perform a study similar to that which you did in Homework #8. What conclusions do you reach?

And as always, email your code to my yahoo address. Please send me **just one m-file**, in which the code for **all four methods** appears.