MATH 566 – Homework #8

due 30 October 2007

1) Do Exercise 7.5.5

2) Let A be the matrix described by Equation (7.1.10) in the text. Write a computer program in Matlab to solve

$$A\mathbf{x} = \mathbf{b},$$

via the steepest descent method in both its unpreconditioned and preconditioned versions. Your preconditioner is the matrix

$$M = \begin{bmatrix} T & & & \\ & T & & \\ & & \ddots & \\ & & & T \end{bmatrix}.$$

Your program should read the data from a file called `hw8.dat` which must be organized in this order:

- the positive integer m that defines the size of the problem
- the maximum number of iterations permitted
- the "tolerance", i.e., the positive number ϵ such that the program will terminate when $\|\mathbf{r}\| < \epsilon$
- the entries of vector $\mathbf{b}$, starting from the top
- the entries of the "initial guess" vector $\mathbf{x}_0$, starting from the top

Recall that in the preconditioned version, we need to compute

$$\mathbf{p} = M^{-1}\mathbf{r}. \tag{1}$$

Of course, we will not compute and store M^{-1}. Describe (in prose) how you solve (1). You want to solve (1) as efficiently as possible. What options occur to you?

When you get both versions working properly, we should see which one is superior. Of course, the preconditioned version should converge in fewer iterations, but since each iteration is more expensive, which is truly the faster method?

Investigate this question by fixing ϵ and running the code for varying values of m, then fix m and then run the code for varying values ϵ. For these investigations, let the initial guess vector equal the zero vector and let the vector $\mathbf{b}$ be such that the entries of the solution vector $\mathbf{x}$ are $x_i = (-1)^i$. What conclusions do you draw?

Matlab's `clock` command should come in handy.

And as always, email your code to my yahoo address.