MATH 566 – Homework #6

due 18 October 2007

1) Write a computer program in Matlab to solve

\[ Ax = b, \]

where \( A \) is \( n \times n \), via the Jacobi method.

Your program should read the data from a file called \texttt{hw6JGS.dat} which must be organized in this order:

- the size \( n \) of the problem
- the maximum number of iterations permitted
- the non-zero entries of matrix \( A \), one row at a time, starting with the top row
- all entries of vector \( b \), starting from the top

2) Write a computer program in Matlab to solve

\[ Ax = b, \]

where \( A \) is \( n \times n \), via the Gauss-Seidel method.

Your program should read the data from the file \texttt{hw6JGS.dat} (i.e., the same data file as you used for the Jacobi method).

3) Write a computer program in Matlab to solve

\[ Ax = b, \]

where \( A \) is \( n \times n \), via the SOR method.

Your program should read the data from a file called \texttt{hw6SOR.dat} which must be organized in this order:

- the size \( n \) of the problem
- the maximum number of iterations permitted
- the SOR parameter \( \omega \)
- the non-zero entries of matrix \( A \), one row at a time, starting with the top row
• all entries of vector \( b \), starting from the top

4) Consider the \( n \times n \) tridiagonal matrix

\[
A = \begin{bmatrix}
-2 & 1 & 0 & 0 & \cdots & 0 \\
1 & -2 & 1 & 0 & \cdots & 0 \\
0 & 1 & -2 & 1 & \cdots & 0 \\
& \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & 1 & -2 & 1 \\
0 & \cdots & 0 & 0 & 1 & -2 \\
\end{bmatrix}
\]

a) Consider Richardson’s method to solve

\[
Ax = b.
\]

How should \( \omega \) be chosen so that \( M \) approximates \( A \) as well as possible?

b) Find formulas for the eigenvalues of \( G = M^{-1} N \).

c) Write a computer program to solve

\[
Ax = b.
\]

Demonstrate, using output from your program, that the estimate

\[
\frac{\|e^{(k+1)}\|}{\|e^{(k)}\|} \approx \rho(G) \tag{1}
\]

holds for sufficiently large \( k \). The purpose of your computer code is to demonstrate the estimate (1), not to write a lovely implementation of Richardson’s method.

5) Let \( A \) be the real \( 2 \times 2 \) matrix

\[
A = \begin{bmatrix}
a & b \\
c & d \\
\end{bmatrix}
\]

and consider using both the Jacobi and Gauss-Seidel methods to solve

\[
Ax = b
\]

for \( x \). Prove it is impossible to have the following conditions all satisfied simultaneously:

• the Jacobi method converges
• the Gauss-Seidel method converges
• the Jacobi method converges more quickly than does the Gauss-Seidel method