# MATH 433/533 – Final Exam

## due 11 May 2009 at 1500

You must show all your work to receive full credit. You may not collaborate with others. GOOD LUCK!

1) Find the general solution of

$$x^2y' + 2xy = y^3.$$

2) Find the general solution of

$$x^4u'' + 7x^3u' + 9x^2u = \ln x.$$

3) Consider the differential equation

$$y'' + 2y' + q(x)y = r(x), \tag{1}$$

where

$$q(x) = \begin{cases} 1 \text{ if } x > 0 \\ -8 \text{ if } x < 0. \end{cases}$$

a) Determine explicitly a basis of solutions of (1) when $r(x) = 0$. Your basis functions must be in $C^1$.

b) Find the general solution of (1) when $r(x) = \cos x$.

c) Let $y_p(x)$ be the particular solution you found in part b). Compute $y_p'(x)$ and $y_p''(x)$.

d) Show that there is complete balance of discontinities when substituting $y_p(x)$, $y_p'(x)$ and $y_p''(x)$ into the DE (1).

4) Problems 4, 6, 7 from Exercises D, p. 57

5) Consider the boundary value problem

$$\begin{cases} u''' + \lambda u = 0 \\ u(0) = 0 \\ u'(0) = 0 \\ u(\pi) = 0. \end{cases} \tag{2}$$

Find an expression that determines all eigenvalues $\lambda$ of (2). Then determine numerical approximations for the smallest three positive eigenvalues.

6) Do the part of Problem 1(c) from Exercises A, p. 304, that says: show that the given SL system has an infinite sequence of positive eigenvalues.

7) Problem 7, Exercises A, p. 305

8) Problem 3, Exercises C, p. 311