You must show all your work to receive full credit. You may not collaborate with others. GOOD LUCK!

1) Find the general solution of
   \[ x^2y' + 2xy = y^3. \]

2) Find the general solution of
   \[ x^4u'' + 7x^3u' + 9x^2u = \ln x. \]

3) Consider the differential equation
   \[ y'' + 2y' + q(x)y = r(x), \tag{1} \]
   where
   \[ q(x) = \begin{cases} 
   1 & \text{if } x > 0 \\
   -8 & \text{if } x < 0.
   \end{cases} \]
   a) Determine explicitly a basis of solutions of (1) when \( r(x) = 0 \). Your basis functions must be in \( C^1 \).
   b) Find the general solution of (1) when \( r(x) = \cos x \).
   c) Let \( y_p(x) \) be the particular solution you found in part b). Compute \( y_p'(x) \) and \( y_p''(x) \).
   d) Show that there is complete balance of discontinuities when substituting \( y_p(x) \), \( y_p'(x) \) and \( y_p''(x) \) into the DE (1).

4) Problems 4, 6, 7 from Exercises D, p. 57
5) Consider the boundary value problem

\[
\begin{align*}
    u''' + \lambda u &= 0 \\
    u(0) &= 0 \\
    u'(0) &= 0 \\
    u(\pi) &= 0.
\end{align*}
\]

Find an expression that determines all eigenvalues \( \lambda \) of (2). Then determine numerical approximations for the smallest three positive eigenvalues.

6) Do the part of Problem 1(c) from Exercises A, p. 304, that says: show that the given SL system has an infinite sequence of positive eigenvalues.

7) Problem 7, Exercises A, p. 305

8) Problem 3, Exercises C, p. 311