MATH 333 – Section 002 – Quiz 3

You may work with other class members on this quiz, but you may not receive assistance from people not in MATH 333 (Section 002). You must show all of your work to receive full credit. Do all your work on other sheets of paper and be sure to staple all the pieces of paper together or YOU WILL GET A ‘ZERO’ ON THE QUIZ. Do not use decimal approximations unless asked to do so. You may not appeal to a table of integrals to assist you. Your work on this quiz must be handed in by Friday, 9 February 2007 at 1340. GOOD LUCK!

1) Consider the initial value problem

\[ \begin{align*}
  y' &= y^{2/3} \\
  y(0) &= 0.
\end{align*} \]

(a) Find two distinct solutions of (1).

(b) Does the fact that you have found two distinct solutions violate the Existence and Uniqueness Theorem (Theorem 2.3.1)? Explain.

2) Consider the initial value problem

\[ \begin{align*}
  y' + \frac{y}{t} &= 1 - 2 \text{ step}(t, 4) \\
  y(0) &= 0.
\end{align*} \]

(a) Solve (2).

(b) Sketch the solution of (2), perhaps using the ODE Architect computer program. If you use ODE Architect, print a hard copy.

(c) Since the right side of the differential equation in (2) has a discontinuity, so must the left side. Display clearly this balance of the discontinuities.

3) Consider the differential equation

\[ y' = ty - y + t - 1. \]

(a) Use the ODE Architect computer program to sketch the slope field of the differential equation (3). Use \(-1 \leq t \leq 4\) and \(-3 \leq y \leq 3\). Print a hard copy.

(b) Where do we have \(y' = 0\)? Answer both mathematically and graphically and show that the two perspectives agree.

(c) On your hard copy, draw by hand the solution to the differential equation (3) corresponding to the initial condition \(y\left(-\frac{1}{2}\right) = 0\).

4) Find the general solution of the differential equation

\[ y' = \frac{y^2}{\csc 2t}. \]

Your answer should give \(y\) as a function of \(t\).