1) Consider the initial value problem

\[
\begin{cases}
y'' + \pi^2 y = \sin(3t) \\
y(0) = 0 \\
y'(0) = (\pi - 3)^{-1}
\end{cases}
\]  

(a) Solve the initial value problem (1).

(b) Manipulate your answer to part a) to demonstrate the existence of slow and fast oscillations. What is the period of each of these oscillations? What is the amplitude of your solution? Give exact answers and appropriate estimates.

(c) Produce a picture of the solution of (1). Draw on your picture to indicate the amplitude, period of fast oscillations, and period of slow oscillations.

2) Consider the initial value problem

\[
\begin{cases}
y'' + \omega_0^2 y = F_0 \cos \omega t \\
y(0) = 0 \\
y'(0) = 0
\end{cases}
\]  

(a) Solve (2) when \(\omega = \omega_0\) and when \(\omega \neq \omega_0\). The result you should obtain is

\[
y(t) = \begin{cases}
\frac{F_0}{2\omega_0} t \sin \omega_0 t & \text{if } \omega = \omega_0 \\
\frac{F_0}{\omega_0^2 - \omega^2} (\cos \omega t - \cos \omega_0 t) & \text{if } \omega \neq \omega_0.
\end{cases}
\]

(b) Based on the result of part a), it would make sense that

\[
\lim_{\omega \to \omega_0} \frac{F_0}{\omega_0^2 - \omega^2} (\cos \omega t - \cos \omega_0 t) = \frac{F_0}{2\omega_0} t \sin \omega_0 t. 
\]  

Prove that (3) is correct.

3) Graph the solution of (2) for \(\omega = \omega_0 = 4\) and \(F_0 = 12\).