MATH 333 – Quiz 9

You may work with other class members on this quiz, but you may *not* receive assistance from people not in MATH 333 (Section 002). You must show all of your work to receive full credit. Do all your work on other sheets of paper and be sure to staple all the pieces of paper together or YOU WILL GET A 'ZERO' ON THE QUIZ. Do not use decimal approximations unless asked to do so. All final solution functions should real-valued. Your work on this quiz must be handed in by Friday, 21 November 2008 at 1040. GOOD LUCK!

1) Consider the initial value problem

$$\begin{cases} y'' + \pi^2 y = \sin(3t) \\ y(0) = 0 \\ y'(0) = (\pi - 3)^{-1} \end{cases} \tag{1}$$

a) Solve the initial value problem (1).

b) Manipulate your answer to part a) to demonstrate the existence of slow and fast oscillations. What is the period of each of these oscillations? What is the amplitude of your solution? Give exact answers and appropriate estimates.

c) Produce a picture of the solution of (1). Draw on your picture to indicate the amplitude, period of fast oscillations, and period of slow oscillations.

2) Consider the initial value problem

$$\begin{cases} y'' + \omega_0^2 y = F_0 \cos \omega t \\ y(0) = 0 \\ y'(0) = 0 \end{cases} \tag{2}$$

a) Solve (2) when $\omega = \omega_0$ and when $\omega \neq \omega_0$. The result you should obtain is

$$y(t) = \begin{cases} \dfrac{F_0}{2\omega_0} t \sin \omega_0 t \text{ if } \omega = \omega_0 \\ \\ \dfrac{F_0}{\omega_0^2 - \omega^2} (\cos \omega t - \cos \omega_0 t) \text{ if } \omega \neq \omega_0. \end{cases}$$

b) Based on the result of part a), it would make sense that

$$\lim_{\omega \to \omega_0} \frac{F_0}{\omega_0^2 - \omega^2} (\cos \omega t - \cos \omega_0 t) = \frac{F_0}{2\omega_0} t \sin \omega_0 t. \tag{3}$$

Prove that (3) is correct.

3) Graph the solution of (2) for $\omega = \omega_0 = 4$ and $F_0 = 12$.