MATH 333 – Quiz 2

You may work with other class members on this quiz, but you may not receive assistance from people not in MATH 333 (Section 002). You must show all of your work to receive full credit. Do all your work on other sheets of paper and be sure to staple all the pieces of paper together or YOU WILL GET A ‘ZERO’ ON THE QUIZ. Do not use decimal approximations unless asked to do so. Your work on this quiz must be handed in by Friday, 12 September 2008 at 1040. GOOD LUCK!

1) Consider the model

\[ \frac{dP}{dt} = P - P^3. \]  

a) Find all equilibrium solutions of (1).
b) Find all values of \( P \) for which is \( P \) increasing.
c) Find all values of \( P \) for which is \( P \) decreasing.

2) Consider the population model

\[ \begin{cases} \frac{dx}{dt} = \alpha x - \beta xy \\ \frac{dy}{dt} = -\gamma y - \delta xy \end{cases} \]

where \( x \) and \( y \) are species populations, \( t \) is time, and \( \alpha, \beta, \gamma, \) and \( \delta \) are positive parameters.

a) Does the presence of species \( x \) help or harm species \( y \)? Explain.
b) Does the presence of species \( y \) help or harm species \( x \)? Explain.
c) According to this model, what happens in the long term to species \( x \) if species \( y \) is extinct? Explain.
d) According to this model, what happens in the long term to species \( y \) if species \( x \) is extinct? Explain.

3) Solve the initial value problem

\[ \begin{cases} \frac{dy}{dt} = \frac{t \sin 2t}{y} \\ y(\pi) = -2 \end{cases} \]

4) An 100-gallon container is half full with pristine water. A solution containing 2 pounds of salt per gallon enters the tank at the rate of 3 gallons per minute while the well-stirred mixture leaves the tank at the same rate.

a) Write an initial value problem that models this process.
b) Solve the initial value problem.