MATH 333 – Section 002 – Quiz 9

You may work with other class members on this quiz, but you may *not* receive assistance from people not in MATH 333 (Section 002). You must show all of your work to receive full credit. Do all your work on other sheets of paper and be sure to staple all the pieces of paper together or YOU WILL GET A 'ZERO' ON THE QUIZ. Do not use decimal approximations unless asked to do so. Your work on this quiz must be handed in by Friday, 10 November 2006 at 1040. GOOD LUCK!

1) Consider the initial value problem

$$\begin{cases} y'' + 2y' - 15y = f(t) \\ y(0) = 1 \\ y'(0) = -2. \end{cases} \tag{1}$$

a) Use convolution and Laplace transforms to solve (1)

b) Use your result in part a) above to solve the initial value problem (1) when

i) $f(t) = 1$

ii) $f(t) = e^{-2t}$

2) Use Laplace transforms to prove:

$$\frac{d}{dt}\left(u_a(t)\right) = \delta(t-a),$$

if $a > 0$.

3) A mass attached to a vertical spring is oscillating, modeled by the initial value problem

$$\begin{cases} y'' + y = k\delta(t-a) \\ y(0) = 0 \\ y'(0) = 1, \end{cases} \tag{2}$$

where y is position (measured in cm, with $y > 0$ indicating that the mass is above its equilibrium position) and t is time (measured in seconds). The δ function models the strike of a hammer on the mass at $t = 7$ seconds. We want the mass to be moving downward at the rate of 0.2 cm/sec when $t = 9$ seconds. How should we choose k to accomplish this goal? Give the exact answer and an appropriate estimate. What are the units of k?