MATH 333 – Section 002 – Quiz 5

You may work with other class members on this quiz, but you may not receive assistance from people not in MATH 333 (Section 002). You must show all of your work to receive full credit. Do all your work on other sheets of paper and be sure to staple all the pieces of paper together or YOU WILL GET A ‘ZERO’ ON THE QUIZ. Do not use decimal approximations unless asked to do so. Your work on this quiz must be handed in by Friday, 6 October 2006 at 1040. GOOD LUCK!

1) Consider the differential equation

\[ y' = y^2 + 4y + c. \]

Perform a bifurcation analysis that culminates in a bifurcation diagram. See Figure 2.9.4 on page 115 for an example of what your diagram should include.

2) 

a) Assume a 40-gram mass attached to a vertical spring is at rest in an equilibrium position. We pull downward on the mass, displacing it by 2 cm, and then release it. We note that the mass first returns to its former equilibrium position 0.3 seconds later. If we assume no damping and no external force, what is the spring constant \( k \)? Give an exact answer and an appropriate estimate.

b) Now that we know \( k \), let’s change the problem a bit. Assume we pull down on the mass, displacing it by \( Q \) cm (with \( Q \) small enough so we do not to destroy the spring), and then release it. When will the mass first return to its former equilibrium position? And what is the significance of this result?