MATH 333 – Section 002 – Quiz 3

You may work with other class members on this quiz, but you may not receive assistance from people not in MATH 333 (Section 002). You must show all of your work to receive full credit. Do all your work on other sheets of paper and be sure to staple all the pieces of paper together or YOU WILL GET A ‘ZERO’ ON THE QUIZ. Do not use decimal approximations unless asked to do so. Your work on this quiz must be handed in by Friday, 15 September 2006 at 1040. GOOD LUCK!

1) Consider the differential equation
\[ y' = y^{2/3}. \]  
(1)

a) Show that the zero function \( y_1(t) = 0 \) satisfies (1).

b) Show that the function \( y_2(t) = \frac{t^3}{27} \) satisfies (1).

c) Show that \( y_1(0) = y_2(0) \) but \( y_1(t) \neq y_2(t) \) for all \( t \).

d) Do the results above contradict Theorem 2.4.1 on page 69? Explain fully.

2) Consider the initial value problem
\[
\begin{cases} 
  y' + 2t^{-1}y = t^2 \text{step}(t, 2) \\
  y(1) = 4.
\end{cases}
\]  
(2)

a) Solve this initial value problem.

b) Is your solution function continuous at \( t = 2 \)? Explain.

c) Is the derivative of your solution function continuous at \( t = 2 \)? Explain.

d) Note that the right side of the differential equation in (2) is discontinuous at \( t = 2 \). So the left side of this differential equation must have a discontinuity that balances the discontinuity on the right side. Demonstrate clearly this balance in the discontinuities.

3) Consider the differential equation
\[ y' = y(1 - y). \]  
(3)

a) Find all equilibrium solutions of (3).

b) Find the general solution of (3), writing \( y \) as a function of \( t \).