

MATH 301 – Quiz 9

You may work with other class members on this quiz, but you may *not* receive assistance from people not in MATH 301, Section 002. You must show all of your work to receive full credit. Do all your work on other sheets of paper and be sure to staple all the pieces of paper together or **YOU WILL GET A 'ZERO' ON THE QUIZ**. Do not use decimal approximations unless asked to do so. Your work on this quiz must be handed in by Friday, 25 April 2008 at the beginning of class. **GOOD LUCK!**

Feel free to use Matlab when appropriate.

1) Determine a 3×3 real matrix A such that

- one eigenvalue is -2 with associated eigenvector $\begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix}$
- another eigenvalue is $3 - 4i$ with associated eigenvector $\begin{bmatrix} -3 + 5i \\ -3i \\ 1 - 6i \end{bmatrix}$

I advise that you use Matlab's format `rat` to obtain your answer. Typing the command `help format` may prove enlightening.

2) Let A be an $n \times n$ matrix and let both B and C be $n \times n$ matrices that are similar to A .

- Is B similar to C ? If yes, prove it. If no, find a counterexample.
- Is BC similar to A ? If yes, prove it. If no, find a counterexample.

(If you find a counterexample, you need to prove your counterexample works!)

3) Let S be the set of all real nonsingular $n \times n$ matrices. Is S a vector space? Prove your answer.

4) Let \mathcal{P}_3 be the vector space of all polynomials of degree 3 or less. Let $\mathbf{v}_1 = x^3 + x^2 + x + 1$, $\mathbf{v}_2 = x^2 + x + 1$, $\mathbf{v}_3 = x + 1$, and $\mathbf{v}_4 = 1$ be elements of \mathcal{P}_3 . Is it possible to find scalars c_1, c_2, c_3 , and c_4 , not all zero, such that $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 + c_4\mathbf{v}_4 = \mathbf{0}$?