MATH 301 – Quiz 8

You may work with other class members on this quiz, but you may not receive assistance from people not in MATH 301, Section 002. You must show all of your work to receive full credit. Do all your work on other sheets of paper and be sure to staple all the pieces of paper together or YOU WILL GET A ‘ZERO’ ON THE QUIZ. Do not use decimal approximations unless asked to do so. Your work on this quiz must be handed in by Friday, 18 April 2008 at the beginning of class. GOOD LUCK!

You may use Matlab to help you compute eigenvectors, but not eigenvalues. Check out Matlab’s eig command, which gives you eigenvalues and eigenvectors. You may use the eig command to check your work, but not to do your work for you.

1) Find all eigenvalues and associated eigenvectors of the matrix

\[
A = \begin{bmatrix}
16 & 8 & -50 \\
-4 & -2 & 20 \\
4 & 2 & -10
\end{bmatrix}.
\]

2) Let \( A \) be a non-singular \( n \times n \) matrix. Let \( \lambda \) be an eigenvalue of \( A \) with associated eigenvector \( \mathbf{x} \). Prove that \( \frac{1}{\lambda} \) is an eigenvalue of \( A^{-1} \) with associated eigenvector \( \mathbf{x} \).

3) Let

\[
A = \begin{bmatrix}
-3 & 3 \\
-3 & -9
\end{bmatrix}.
\]

Find all eigenvalues and associated eigenvectors of \( A \). What is the eigenspace associated with each eigenvalue? What is the algebraic multiplicity and geometric multiplicity of each eigenvalue.

4) Let \( A \) be an \( n \times n \) matrix. Let \( \lambda \) be an eigenvalue of \( A \). Prove that the eigenspace \( E_\lambda \) is a subspace of \( \mathbb{R}^n \).

5) Prove the “Lemmaette”: Let \( A \) be a real \( n \times n \) matrix and let \( \mathbf{x} \) be a vector is \( \mathbb{R}^n \). Prove that

\[
A\mathbf{x} = A\mathbf{x}.
\]