MATH 301 – Quiz 3

You may work with other class members on this quiz, but you may not receive assistance from people not in MATH 301, Section 002. You must show all of your work to receive full credit. Do all your work on other sheets of paper and be sure to staple all the pieces of paper together or YOU WILL GET A ‘ZERO’ ON THE QUIZ. Do not use decimal approximations unless asked to do so. Your work on this quiz must be handed in by Friday, 29 February 2008 at the beginning of class. GOOD LUCK!

1) Let \( A = \begin{bmatrix} 2 & 0 & 3 \\ 1 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix} \). Find \( A^{-1} \).

2) Let \( A = I - a\mathbf{v}\mathbf{v}^T \), where \( \mathbf{v} \) is a non-zero scalar in \( \mathbb{R}^n \) and where \( a = \frac{2}{\mathbf{v}^T \mathbf{v}} \). Prove that \( A \) is symmetric and that \( A^{-1} = A \).

3) Let \( \mathbf{v} \) be a fixed vector in \( \mathbb{R}^n \) and define \( W \) to be the subset of \( \mathbb{R}^n \) given by

\[
W = \{ \mathbf{x} : \mathbf{v}^T \mathbf{x} = 0 \}.
\]

Prove that \( W \) is a subspace of \( \mathbb{R}^n \).

4) Let \( U \) and \( V \) both be subspaces of \( \mathbb{R}^n \). Prove that their intersection \( U \cap V \) is also a subspace of \( \mathbb{R}^n \).