MATH 301 – Final Examination

This take-home part of the exam is due at 1:00 p.m. on 14 May 2003.

1) You are encouraged to used Maple (or similar software) for this problem. In Maple, the symbol ∞ is obtained by typing `infinity` and the symbol e^x is obtained by typing `exp(x)`. You may also encounter strange output. If so, use the command `evalf` to obtain a numerical approximation to the strangeness.

The goal here is to find good polynomial approximations to

$$\phi(x) = x^2 \sin x. \tag{1}$$

a) Use Maple to graph $\phi(x)$ on the interval $[-3, 3]$.

b) Each approximating polynomial is an element of the inner product space $\mathcal{P}_n$, where the inner product is given by

$$\langle f, g \rangle = \int_{-\infty}^{\infty} e^{-x^2} f(x)\, g(x)\, dx. \tag{2}$$

The function e^{-x^2} is known as a *weighting function*. Create an othogonal basis for $\mathcal{P}_5$, where orthogonality is determined in terms of the inner product (2). The elements in your orthogonal basis are the *Hermite polynomials*.

c) For each $j = 0, 1, 2, 3, 4, 5$, determine the best least squares approximation p_j^* for (1), where each p_j^* is in the inner product space $\mathcal{P}_j$ with inner product (2). For each j, plot (1) and p_j^* on the same set of axes.

d) Plot, on the same set of axes, *all* of your polynomial approximations p_j^* as well as the function (1). Write a brief essay about your observations with respect to this picture and this entire take-home exercise.