

MATH 275 – Section 002 – Quiz 9

You may work with other class members on this quiz, but you may *not* receive assistance from people not in MATH 275 (Section 002). You must show all of your work to receive full credit. Do all your work on other sheets of paper and be sure to staple all the pieces of paper together or YOU WILL GET A 'ZERO' ON THE QUIZ. Do not use decimal approximations unless asked to do so. Your work on this quiz must be handed in by Friday, 20 April 2007 at 1440. GOOD LUCK!

1) Evaluate:

$$\iint_R x^2 y \, dA,$$

where R is the region in Quadrant I bounded by $y = \frac{1}{x}$, $y = \frac{4}{x}$, $y = x$, and $y = 2x$.

2) A fence is built so its base lies along the circle

$$x^2 + y^2 = 16$$

from the point $(4, 0)$ to the point $(\sqrt{12}, 2)$. The height of the fence at each point (x, y) of the circular arc is given by

$$f(x, y) = x + y.$$

Determine the area of the fence.

3) Consider the velocity field given by

$$\mathbf{F}(x, y) = \begin{bmatrix} y \\ -x \end{bmatrix}.$$

- Draw a picture that represents this velocity field.
- Let C be the ellipse given by $x^2 + 4y^2 = 1$, traversed in the counter-clockwise direction. Add this ellipse to your picture.
- Based *only on your picture*, do you expect that the circulation of \mathbf{F} around C is positive or negative? Explain.
- Calculate the circulation of \mathbf{F} around C .

4) Consider the velocity field given by

$$\mathbf{G}(x, y) = \begin{bmatrix} -\frac{1}{3}x \\ -\frac{2}{3}y \end{bmatrix}.$$

- a) Draw a picture that represents this velocity field.
- b) Let C be the circle given by $x^2 + y^2 = 9$, traversed in the counter-clockwise direction. Add this circle to your picture.
- c) Based *only on your picture*, do you expect that the flux of \mathbf{G} across C is positive or negative? Explain.
- d) Calculate the flux of \mathbf{G} across C .