MATH 275 – Section 002 – Quiz 4

You may work with other class members on this quiz, but you may not receive assistance from people not in MATH 275 (Section 002). You must show all of your work to receive full credit. Do all your work on other sheets of paper and be sure to staple all the pieces of paper together or YOU WILL GET A ‘ZERO’ ON THE QUIZ. Do not use decimal approximations unless asked to do so. Your work on this quiz must be handed in by Monday, 9 February 2004 at 12:40 p.m. GOOD LUCK!

1) Consider the ellipse in the \(x-y\) plane whose equation is
\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.
\]
Find a parametrization of this ellipse using trigonometric functions. What range of values of the parameter generates the complete ellipse exactly once?

2) Consider the intersection of the sphere \(x^2 + y^2 + z^2 = 1\) and the plane \(x = -2z\). Your geometric intuition should tell you that these two surfaces intersect in a circle.

   a) Find a parametrization of the curve of intersection. Use your result from Exercise 1) above to accomplish this. What range of values of the parameter generates the complete curve of intersection exactly once?

   b) If your curve of intersection in part a) is truly a circle, then every point on the curve is a constant distance (the radius) from a particular point (the center). Identify the center and radius and prove that every point on your curve of intersection is this constant distance from the center.

3) Find a vector function \(\mathbf{r}(t)\) such that \(\mathbf{r}'(t) = \frac{1}{t} \mathbf{i} + \frac{1}{1+t^2} \mathbf{j} + \sin(2t) \mathbf{k}\) and \(\mathbf{r}(1) = 3 \mathbf{i} + \frac{\pi}{2} \mathbf{j}\).