MATH 275 – Section 002 – Quiz 11

You may work with other class members on this quiz, but you may not receive assistance from people not in MATH 275 (Section 002). You must show all of your work to receive full credit. Do all your work on other sheets of paper and be sure to staple all the pieces of paper together or YOU WILL GET A ‘ZERO’ ON THE QUIZ. Do not use decimal approximations unless asked to do so. Your work on this quiz must be handed in by Monday, 19 April 2004 at 12:40 p.m. GOOD LUCK!

1) A thin wire is bent into the shape of the helix given by

\[ \mathbf{r}(t) = \begin{bmatrix} \cos 2t \\ \sin 2t \\ t \end{bmatrix} \]

for \(0 \leq t \leq \frac{\pi}{2}\). The density function for the wire is \(\delta(x, y, z) = x + y\). Find the center of mass of the wire.

2) You may \textbf{NOT} use Green’s Theorem on this problem.

Let a velocity field be given by

\[ \mathbf{F}(x, y) = \begin{bmatrix} -y \\ -y \end{bmatrix} \]

and let \(C\) be the unit circle in the \(x\)-\(y\) plane with center at the origin, traversed counterclockwise.

a) Draw a picture which shows both the vector field \(\mathbf{F}\) and the curve \(C\).

b) Based only on your picture, determine whether the circulation of \(\mathbf{F}\) around \(C\) is positive or negative. Explain your reasoning.

c) Based only on your picture, determine whether the flux of \(\mathbf{F}\) across \(C\) is positive or negative. Explain your reasoning.

d) Calculate both the circulation and the flux. Do your calculations agree with your conclusions from parts b) and c)? Explain.