MATH 275 – Section 001 – Quiz 4

You may work with other class members on this quiz, but you may not receive assistance from people not in MATH 275 (Section 001). You must show all of your work to receive full credit. Do all your work on other sheets of paper and be sure to staple all the pieces of paper together or YOU WILL GET A ‘ZERO’ ON THE QUIZ. Do not use decimal approximations unless asked to do so. Your work on this quiz must be handed in by Monday, 23 September 2002 at 9:40 a.m. GOOD LUCK!

1) The point \((-\sqrt{3}, -1, 2)\) is given in rectangular coordinates. Convert this point to cylindrical and spherical coordinates.

2) The point \((2, -\frac{\pi}{6}, -3)\) is given in cylindrical coordinates. Convert this point to rectangular and spherical coordinates.

3) The point \((4\sqrt{3}, \frac{5\pi}{4}, \frac{2\pi}{3})\) is given in spherical coordinates. Convert this point to rectangular and cylindrical coordinates.

4) Prove:
\[
\frac{d}{dt}[\mathbf{x}(t) \cdot \mathbf{y}(t)] = \mathbf{x}(t) \cdot \mathbf{y}'(t) + \mathbf{y}(t) \cdot \mathbf{x}'(t)
\]
where \(\mathbf{x}(t)\) and \(\mathbf{y}(t)\) are vectors of three components each.

5) Let \(\mathbf{r}(t) = \begin{bmatrix} 4t \\ \cos 3t \\ \sin 3t \end{bmatrix}\). Find the length of the curve traced out by \(\mathbf{r}\) from \(t = 0\) to \(t = \frac{\pi}{2}\).

6) Let \(\mathbf{r}(t) = \begin{bmatrix} 1 \\ t \\ t^2 \end{bmatrix}\). Find the curvature when \(t = 3\).

7) Let \(\mathbf{r}(t) = \begin{bmatrix} \sin 8t \\ 6t \\ \cos 8t \end{bmatrix}\).
   a) Find the vectors \(\mathbf{T}, \mathbf{N},\) and \(\mathbf{B}\) when \(t = 0\).
   b) Find equations for the normal plane and osculating plane for \(\mathbf{r}(t)\) at \(t = 0\).