MATH 271 – Homework #9

due 2 April 2009

All calculations must be done in Matlab!

1) Modify your Euler's method code from last time to prompt the user for the beginning and ending values for t, the beginning value of y, and the number of time steps.

2) Let

$$A = \begin{bmatrix} 1 & 1 \\ -4 & 1 \end{bmatrix}.$$

Solve the initial value problem

$$\begin{cases} \mathbf{x}' = A\mathbf{x} \\ \mathbf{x}(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \end{cases}$$

using Matlab's `ode45` function, for $0 \le t \le \pi$. Create Matlab figures to compare the numerical solution to the exact solution, which is

$$\mathbf{x}(t) = e^t \begin{bmatrix} \cos 2t + \sin 2t \\ 2\cos 2t - 2\sin 2t \end{bmatrix}.$$

3) Consider the initial value problem

$$\begin{cases} u'' + (\cos x)u' + (\sin x)u = 1 - \sin x \\ u(0) = 0 \\ u'(0) = 1. \end{cases} \tag{1}$$

Convert (1) to a system of two first-order differential equations with their corresponding initial conditions. Then use Matlab's `ode45` function to solve this first-order system for $0 \le t \le 2\pi$. Create Matlab figures to compare the numerical solution to the exact solution, which is $u(x) = \sin x$.