1) Modify your Euler’s method code from last time to prompt the user for the beginning and ending values for $t$, the beginning value of $y$, and the number of time steps.

2) Let

$$ A = \begin{bmatrix} 1 & 1 \\ -4 & 1 \end{bmatrix}. $$

Solve the initial value problem

$$ \begin{cases} x' = Ax \\ x(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \end{cases} $$

using Matlab’s $\texttt{ode45}$ function, for $0 \leq t \leq \pi$. Create Matlab figures to compare the numerical solution to the exact solution, which is

$$ x(t) = e^t \begin{bmatrix} \cos 2t + \sin 2t \\ 2 \cos 2t - 2 \sin 2t \end{bmatrix}. $$

3) Consider the initial value problem

$$ \begin{cases} u'' + (\cos x)u' + (\sin x)u = 1 - \sin x \\ u(0) = 0 \\ u'(0) = 1. \end{cases} \quad (1) $$

Convert (1) to a system of two first-order differential equations with their corresponding initial conditions. Then use Matlab’s $\texttt{ode45}$ function to solve this first-order system for $0 \leq t \leq 2\pi$. Create Matlab figures to compare the numerical solution to the exact solution, which is $u(x) = \sin x$. 