MATH 187 – Section 002 – Quiz 3

You may work with other class members on this quiz, but you may not receive assistance from people not in MATH 187, Section 002. You must show all of your work to receive full credit. Do all your work on other sheets of paper and be sure to staple all the pieces of paper together or YOU WILL GET A ‘ZERO’ ON THE QUIZ. Your work on this quiz must be handed in by the beginning of class on Friday, 15 February 2008. GOOD LUCK!

It is very important that you write down how you are thinking about these exercises. If you get them wrong and haven’t indicated how you are thinking about them, then I’ll have a hard time awarding you any partial credit.

Also, I do not know how to produce some of the symbols we have been using. For example, on this quiz (and future quizzes), $\mathbb{Z}$ represents the set of integers.

1) Write out the following sets by listing their elements between curly braces.
   a) $P = \{S : S \subseteq \{2, 4, 6\} \text{ and } 4 \not\in S\}$
   b) $Q = \{y : y \subseteq \{1, 3, 5\} \text{ and } |y| \geq 2\}$

2) Simplify the following expressions.
   a) $2^{2^{2^1}}$
   b) $2|a^3|$
   c) $|\{\{1, 2\}, \{2, 3\}|$

3) Let $A = \{x \in \mathbb{Z} : x|3\}$ and let $B = \{x \in \mathbb{Z} : x|12\}$. Prove that $B \subseteq A$.

4) Let $A$ be the statement “Every teddy bear in my room is green.” Let $B$ be the statement “At least one of the books in my room is about botany.”
   a) Write statements $A$ and $B$ using the quantifiers $\exists$ and $\forall$.
   b) Write the statement $\neg A$ in two ways: one using $\exists$ and one using $\forall$.
   c) Write the statement $\neg B$ in two ways: one using $\exists$ and one using $\forall$.

5) Consider the statements $A$ and $B$, where $A = “\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, xy > 100”$ and where $B = “\exists y \in \mathbb{Z}, \forall x \in \mathbb{Z}, xy > 100”$.
   a) Is $A$ true? Explain.
   b) Is $B$ true? Explain.