1) \( A \cap B \cap C = \emptyset \)

Let \( A = \{1, 2\} \), \( B = \{2, 3\} \), and \( C = \{3, 4\} \). Then \( A \cap B \cap C = \emptyset \)

So \( A \cup B \cup C = \{1, 2, 3, 4\} \) so \( |A \cup B \cup C| = 4 \)

But \( |A| = |B| = |C| = 2 \)

So \( |A| + |B| + |C| = 6 \)

So \( |A \cup B \cup C| + |A| + |B| + |C| \), so the statement is false.

2) \( 4 \) men, \( 4 \) women

a) \[
\begin{array}{cccc}
8 & 4 & 3 & 3 & 2 & 2 & 1 & 1 \\
\end{array}
\]

\[
8 \cdot 4 \cdot 3^2 \cdot 2^2 = 1152
\]

b) Consider the lines of four people.

\( A, B, C, D \) form the same circle \( A, B, C, D \).

So if \( N \) is the number of lines, then the number of circles is \( \frac{N}{4} \).

So for our example, we want to compute \( \frac{1152}{8} = 144 \)

3) \( A = \{1\} \)

a) \( 2^A = \{\emptyset, A\} \)

b) \( 2^{2^A} = \{\emptyset, \emptyset, \emptyset, \emptyset, \{A\}, 2^A\} \)
4) \( \varnothing = \{ \varnothing \} \) is false

\[ |\varnothing| = 0 \quad \text{but} \quad |\{\varnothing\}| = 1. \]

Since their cardinalities are different, \( \varnothing \) and \( \{\varnothing\} \) cannot be the same set.

5) Prove \( A \subseteq B \) iff \( A - B = \varnothing \)

\textbf{PF} Need to show

1) If \( A \subseteq B \), then \( A - B = \varnothing \)

and

2) If \( A - B = \varnothing \), then \( A \subseteq B \)

\textbf{pf of 1} It is equivalent to show that if \( A - B \neq \varnothing \), then \( A \nsubseteq B \).

So let \( A - B \neq \varnothing \). So \( \exists x \text{ st } x \in A - B \).

So \( x \in A \) and \( x \notin B \). So \( A \nsubseteq B \). Done with 1.

\textbf{pf of 2} It is equivalent to show that if \( A \nsubseteq B \), then \( A - B \neq \varnothing \).

So let \( A \nsubseteq B \). Then \( \exists x \text{ st. } x \in A \) and \( x \notin B \).

So \( x \in A - B \). So \( A - B \neq \varnothing \). Done with 2. \( \Box \)
6) **reflexive**  Is \( xRx \) true for all \( x \in S \)?

\( xRx \) means \( f(x) \neq f(x) \). But \( f(x) \) does equal \( f(x) \)
\( \forall x \in S \). Thus \( R \) is not reflexive.

**symmetric**  Let \( xRy \), must we have \( yRx \)?

Since \( xRy \), we have \( f(x) \neq f(y) \)
so \( f(y) \neq f(x) \)
so \( yRx \). Thus \( R \) is symmetric.

**transitive**  Let \( xRy \) and \( yRz \), must we have \( xRz \)?

Counterexample. Let \( x = 123 \). Then \( f(x) = 1+2-3 = 0 \)
Let \( y = 124 \) \( f(y) = 1+2-4 = -1 \)
Let \( z = 213 \) \( f(z) = 2+1-3 = 0 \)

So \( f(x) \neq f(y) \) and \( f(y) \neq f(z) \) and \( f(x) = f(z) \)
so \( xRy \) and \( yRz \) and \( xRz \).

So \( R \) is not transitive.