MATH 175 – Section 004 – Quiz 12

You may work with other class members on this quiz, but you may *not* receive assistance from people not in MATH 175 (Section 004). You must show all of your work to receive full credit. Do all your work on other sheets of paper and be sure to staple all the pieces of paper together or YOU WILL GET A 'ZERO' ON THE QUIZ. Do not use decimal approximations unless asked to do so. You may use Maple only when explicit permission is given. Your work on this quiz must be handed in by Monday, 1 December 2003 at 1:40 p.m. GOOD LUCK!

1) **Please use Maple for this exercise. Obtain decimal approximations when appropriate.** Consider the series

$$\sum_{i=1}^{\infty} i^{-4}. \tag{1}$$

Suppose we wish to approximate (1) using

$$\sum_{i=1}^{n} i^{-4}.$$

a) How large should n be so that the error of our approximation is less than 0.001? Use the technique presented in class which, unfortunately, is not discussed completely in the text. A terse example of this technique does appear in the solutions manual: see exercise 33, section 11.3. If you look at the solutions manual, you see that

$$n > 13.08$$

appears somewhat magically. How was this obtained? Use the Maple command

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solve(n^(-1/2) - (n+1)^(-1/2) > 0.01);
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which gives a numerical solution to the equation

$$n^{(-1/2)} - (n+1)^{(-1/2)} > 0.01.$$

Note that the answer that Maple provides here is an *interval*.

b) Use Maple to verify that your answer to part a) does indeed achieve the desired accuracy. Use formula 3 on the bottom of page 718.

2) Determine whether the following series converge or diverge and prove your answer.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n^{3/2}+\frac{1}{n}}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n + \arctan n}{3^n - 1}$

c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{(n+1)} \cos\left(\frac{1}{n}\right)$

3) Consider the series

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots \tag{2}$$

a) Prove this series converges.

b) Suppose we want to estimate (2) so that the error of the estimation is less than 0.01. How many terms of (2) should we use to achieve this accuracy? Use the Alternating Series Estimation Theorem on page 729.

c) Use Maple to verify that your approximation does indeed achieve this degree of accuracy.